ALGEBRA READINESS DIAGNOSTIC TEST PRACTICE

Directions: Study the examples, work the problems, then check your answers at the end of each topic. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands the topic.

TOPIC 1: INTEGERS

A. <u>What is an integer</u>?

Any natural number (1, 2, 3, 4, 5,…), its opposite (-1,-2, -3, -4, -5,…), or zero (0). (Integers are useful for problems involving "below normal," debts, "below sea level," etc.)

Problems 1-10: Identify each number as an integer (I) or not an integer (NI):

1. 367	6. 0
2. -4.4	7. $-\frac{2}{3}$
3. $2\frac{1}{2}$	8. 0.027
4. –1010	9. $\frac{1}{2}$
5. $\sqrt{100}$	10. 2^3

Problems 11-14: Write the opposite of each integer:

11. 42	13. 0
12. -3	14. -4^3

Problems 15-19: Choose the greater:

15. 5, –10	18. –5, 0
16. 5,–5	19. –5,–10
17. 5, 0	

20. What is the result of adding an integer and its opposite?
21. What number is its own opposite?

B. <u>Absolute Value</u>:

Absolute value is used for finding distance, explaining addition of integers, etc.
The absolute value of a positive number or zero is itself. The absolute value of a negative number is its opposite.

Problems 22-26: Choose the integer with the greater absolute value:

22. 4 or –3	25. 3 or 0
23. –4 or 3	26. – 3 or 0
24. 3 or –3	

C. <u>Adding, subtracting, multiplying and dividing integers</u>:

<u>To add two integers</u>:

<u>Both positive</u>: add as natural numbers:
example: Add 4 and 3: $4+3=7$

<u>Both negative</u>: add as though positive; make the result negative:
example: Add –4 and –3:
Treat as positive and add: $4+3=7$.
The answer is –7 because it must be negative.

<u>One positive, one negative</u>: treat each as positive, subtract, make the answer sign of the one with the greater absolute value:
example: Add –4 and 3: $4-3=1$; the answer is –1 because –4 has the greater absolute value.
example: Add 4 and –3: $4-3=1$; the answer is 1 because 4 has the greater absolute value.

Problems 27-33: Add the two integers:

27. 4 and –3 (This means $(4)+(-3)$)

28. 4 and 3	31. –4 and 3
29. –4 and –3	32. 16 and –7
30. 4 and 0	33. –3 and 0

<u>To subtract two integers</u>: add the opposite of the one to be subtracted:
example: 3 subtract –4, or $(3)-(-4)$: The opposite of –4 is 4, so we <u>add</u> 4 (rather than subtract –4). We change the problem from $(3)-(-4)$ to $(3)+(4)$, which we know how to do: $(3)+(4)=3+4=7$
example: $(-4)-(3)$: Add the opposite of 3, namely –3: $(-4)-(3)=(-4)+(-3)=-7$
example: $(4)-(3)=(4)+(-3)=1$
example: $-5-8=(-5)-(8)=(-5)+(-8)=-13$

Problems 34-43: Calculate:

34. $(12)-(3)=$
35. $-12-3=$ (Hint: this means $-12+(-3)$)

36. $-12-(-3)=$	40. $0-3=$
37. $3-12=$	41. $0+4=$
38. $-3-12=$	42. $-12+3=$
39. $(-7)-(-7)=$	43. $(-3)+(-12)=$

<u>To multiply two integers</u>:

1st integer ×	2nd integer =	Answer
+	+	+
−	+	−
+	−	−
−	−	+

<u>Both positive</u>: multiply as two natural numbers.
example: $(3) \times (4) = 3 \times 4 = 12$

<u>Both negative</u>: multiply as if positive; and make the answer positive. and remember, two negatives make a positive. When multiplying two negative numbers, you always get a positive answer.
example: $(-3)(-4)$ so $3 \times 4 = 12$; make it positive, and the answer is 12.

<u>One positive, one negative</u>: When multiplying a negative number and a positive number, the answer is always negative.
example: $(3)(-4)$ so $3 \times 4 = 12$; make the answer negative; answer –12.

Problems 44-55: Multiply:

44. $3 \times (-4) =$	50. $(-4) \bullet 0 =$
45. $(3) \bullet (-4) =$	51. $0^2 =$
46. $(3)(-4) =$	52. $(-3)^2 =$
47. $3(-4) =$	53. $(4)^2 =$
48. $(-3)(-4) =$	54. $(-3) \bullet 4 =$
49. $-3(-4) =$	55. $3 \bullet 4 =$

<u>Reciprocals</u> are used for dividing. Every integer except zero has a reciprocal. The reciprocal is the number that multiplies the integer to give 1.

example: $6 \bullet \frac{1}{6} = 1$, so the reciprocal of 6 is $\frac{1}{6}$.
(And the reciprocal of $\frac{1}{6}$ is 6.)
example: $(-4)\left(-\frac{1}{4}\right) = 1$, so the reciprocal of –4 is $-\frac{1}{4}$.

Problems 56-59: Find the reciprocal:

56. –5	57. 1	58. 10	59. –1

60. What number is its own reciprocal? (Can you find "more than one"?)
61. Using the reciprocal definition, explain why there is no reciprocal of zero.

<u>To divide two integers</u>: multiply by the reciprocal of the one to be divided by:
example: 20 divided by $-5 = 20 \div (-5)$.
The reciprocal of –5 is $-\frac{1}{5}$ so we multiply by $-\frac{1}{5}$: $20 \div (-5) = 20 \times \left(-\frac{1}{5}\right) =$
$\frac{20}{1} \times \left(-\frac{1}{5}\right) = -\frac{20 \bullet 1}{1 \bullet 5} = -\frac{20}{5} = -4$
example: $\frac{-5}{20} = -5 \div 20 = -5 \bullet \frac{1}{20} = -\frac{1}{4}$
example: $\frac{-3}{-6} = -3 \div (-6) = -3 \bullet \left(-\frac{1}{6}\right) = \frac{3}{6} = \frac{1}{2}$
(Note negative times negative is positive.)

example: $\frac{0}{3} = 0 \div 3 = 0 \bullet \frac{1}{3} = 0$

Problems 62-67: Calculate:

62. $(-14) \div (-2) =$	65. $\frac{-15}{3} =$
63. $2 \div 3 =$	66. $\frac{-5}{0} =$(careful)*
64. $3 \div 2 =$	67. $\frac{0}{7} =$

* Problem 61 says $\frac{1}{0}$ has no value (you cannot divide by zero).

68. From the rule for division, why is it impossible to divide by zero?

<u>To "sum" it all up:</u>
Positive+positive = larger positive
Negative+negative = more negative
Positive+negative = in between both

Positive × positive = positive
Negative × negative = positive
Positive × negative = negative

To subtract add the opposite.
To divide, multiply by the reciprocal.

69. Given the statement "Two negatives make a positive." Provide an example of a situation where the statement would be true and another when it would be false.
70. Write "18 divided by 30" in three ways: using ÷, $\overline{)\ }$, and using a fraction bar —.

Problems 71- 80: Calculate:

71. $4 - 10 + 3 - 2 =$	76. $-2[(-6)(8) + 9] =$
72. $4 - (10 + 3 - 2) =$	77. $5 + (3 - 7) =$
73. $4 + 3 - (10 - 2) =$	78. $5 - (3 - 7) =$
74. $6(8 - 3) =$	79. $5 - 3 + 7 =$
75. $(-6)(8) + 9 =$	80. $-1 + 2 - 3 + 4 =$

81. What is the meaning of "sum", "product", quotient", and "difference"?

D. <u>Factoring</u>:
If a number is the product of two (or more) integers, then the integers are factors of the number.

example: $40 = 4 \times 10$, 2×20, 1×40, and 8×5.
So 1, 2, 4, 5, 8, 10, 20, 40 are all factors of 40.
(So are all their negatives.)

Problems 82-86: Find all positive factors of:

82. 10	83. 7	84. 24	85. 9	86. 1

If a positive integer has exactly two positive factors, it is a <u>prime number</u>. Prime numbers are used to find the greatest common factor (GCF) and least common multiple (LCM), which are used to reduce fractions and find common denominators, which in turn are often needed for adding and subtracting fractions.

example: The only positive factors of 7 are 1 and 7, so 7 is a prime number.
example: 6 is not prime, as it has 4 positive factors: 1, 2, 3, 6.

87. From the prime number definition, why is 1 *not* a prime?
88. Write the 25 prime numbers from 1 to 100.

Every positive integer has one way it can be factored into primes, called its prime factorization.

example: Find the prime factorization (PF) of 30: $30 = 3 \times 10 = 3 \times 2 \times 5$, so the PF of 30 is $2 \bullet 3 \bullet 5$.
example: $72 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 3 = 2^3 \bullet 3^2$, the PF. (The PF can be found by making a "factor tree.")

Problems 89-91: Find the PF:

89. 36
90. 10
91. 7

Greatest common factor (GCF) and least common multiple (LCM). If you need to review GCF or LCM, see the worksheet in this series: "Topic 2: Fractions".

Problems 92-95: Find the GCF and the LCM of:

92. 4 and 6
93. 4 and 7
94. 4 and 8
95. 3 and 5

E. Word problems:

96. The temperature goes from $-14°$ to $28°$C. How many degrees Celsius does it change?
97. $28 - (-14) =$
98. Derek owes \$43, has \$95, so "is worth"…?
99. If you hike in Death Valley from 282 feet below sea level to 1000 feet above sea level, how many feet of elevation have you gained?
100. $1000 - (-282) =$
101. A hike from 243 feet below sea level (FBSL) to 85 FBSL means a gain in elevation of how many feet?
102. $-85 - (-243) =$
103. What number added to –14 gives –24?
104. What does "an integral number" mean?
105. Jim wrote a check for \$318. His balance is then \$2126. What was the balance before he wrote the check?
106. What number multiplied by 6 gives –18?
107. If you hike downhill and lose 1700 feet of elevation and end at 3985 feet above sea level (FASL), what was your starting elevation?
108. Anne was 38 miles south of her home. She drove 56 miles north. How far from home was she at that time and in what direction?
109. 5 subtracted from what number gives –12?
110. What number minus negative four gives ten?

Answers:

1. I
2. NI
3. NI
4. I
5. I
6. I
7. NI
8. NI
9. NI
10. I
11. –42
12. 3
13. 0
14. 64
15. 5
16. 5
17. 5
18. 0
19. –5
20. zero
21. zero
22. 4
23. –4
24. both same
25. 3
26. –3
27. 1
28. 7
29. –7
30. 4
31. –1
32. 9
33. –3
34. 9
35. –15
36. –9
37. –9
38. –15
39. 0
40. –3
41. 4
42. –9
43. –15
44. –12
45. –12
46. –12
47. –12
48. 12
49. 12
50. 0
51. 0
52. 9
53. 16
54. –12
55. 12
56. $-\frac{1}{5}$
57. 1
58. $\frac{1}{10}$
59. –1
60. 1; also –1
61. no number times $0 = 1$
62. 7
63. $\frac{2}{3}$
64. $\frac{3}{2}$

65. –5
66. no value (not defined)
67. 0
68. zero has no reciprocal
69. true if $\times$, false if $+$.
70. $18 \div 30$, $30\overline{)18}$, 18/30
71. –5
72. –7
73. –1
74. 30
75. –39
76. 78
77. 1
78. 9
79. 9
80. 2
81. $+$, $\times$, $\div$, $-$
82. 1, 2, 5, 10
83. 1, 7
84. 1, 2, 3, 4, 6, 8, 12, 24
85. 1, 3, 9
86. 1
87. 1 has one factor
88. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
89. $2^2 \bullet 3^2$
90. $2 \bullet 5$
91. 7
92. 2, 12
93. 1, 28
94. 4, 8
95. 1, 15
96. 42
97. 42
98. $52
99. 1282
100. 1282
101. 158
102. 158
103. –10
104. an integer
105. $2444
106. –3
107. 5685 FASL
108. 18 mi. N
109. –7
110. 6

TOPIC 2: FRACTIONS

A. <u>Greatest Common Factor (GCF)</u>:
The GCF of two integers is used to simplify (reduce, rename) a fraction to an equivalent fraction. A <u>factor</u> is an integer <u>multiplier</u>. A <u>prime number</u> is a positive whole number with exactly two positive factors.

example: the prime factorization of 18 is $2 \bullet 3 \bullet 3$, or $2 \bullet 3^2$.

Problems 1-2: Find the prime factorization:

1. 24
2. 42

example: Find the factors of 42.
Factor into primes: $42 = 2 \bullet 3 \bullet 7$
1 is always a factor
2 is a prime factor
3 is a prime factor
4 is a prime factor
7 is a prime factor
$2 \bullet 3 = 6$ is a factor
$2 \bullet 7 = 14$ is a factor
$3 \bullet 7 = 21$ is a factor
$2 \bullet 3 \bullet 7 = 42$ is a factor
Thus 42 has 8 factors.

Problems 3-4: Find all positive factors:

3. 18
4. 24

To find the GCF:

example: Looking at the factors of 42 and 24, we see that the common factors of both are 1, 2, 3, and 6, of which the greatest it 6; so: the GCF of 42 and 24 is 6. (Notice that “common factor” means “shared factor.”)

Problems 5-7: Find the GCF of:

5. 18 and 36
6. 27 and 36
7. 8 and 15

B. <u>Simplifying fractions</u>:

example: Reduce $\frac{27}{36}$:
$\frac{27}{36} = \frac{9 \bullet 3}{9 \bullet 4} = \frac{9}{9} \bullet \frac{3}{4} = 1 \bullet \frac{3}{4} = \frac{3}{4}$
(Note that you must be able to find a common factor, in this case 9, in both the top and bottom in order to reduce.)

Problems 8-13: Reduce:

8. $\frac{13}{52} =$
9. $\frac{26}{65} =$
10. $\frac{3+6}{3+9} =$
11. $\frac{16}{64} =$
12. $\frac{24}{42} =$
13. $\frac{24}{18} =$

C. <u>Equivalent Fractions</u>:

example: $\frac{3}{4}$ is equivalent to how many eighths?
$\left(\frac{3}{4} = \frac{\ }{8}\right)$
$\frac{3}{4} = 1 \bullet \frac{3}{4} = \frac{2}{2} \bullet \frac{3}{4} = \frac{2 \bullet 3}{2 \bullet 4} = \frac{6}{8}$

Problems 14-17: Complete:

14. $\frac{4}{9} = \frac{\ }{72}$
15. $\frac{3}{5}$ is how many twentieths?
16. $\frac{56}{100} = \frac{\ }{50}$
17. How many halves are in 3? (Hint: think $3 = \frac{3}{1} = \frac{\ }{2}$)

D. <u>Ratio</u>:
If the ratio of boys to girls in a class is 2 to 3, it means that for every 2 boys, there are 3 girls. A

ratio is like a fraction: think of the ratio 2 to 3 as the fraction $\frac{2}{3}$.

> *example:* If the class had 12 boys, how many girls are there? Write the fraction ratio: $\frac{number\ of\ boys}{number\ of\ girls} = \frac{2}{3} = \frac{12}{}$
>
> Complete the equivalent fraction: $\frac{2}{3} = \frac{2 \bullet 6}{3 \bullet 6} = \frac{12}{18}$
>
> So there are 18 girls.

18. If the class had 21 girls and the ratio of boys to girls was 2 to 3, how many boys would be in the class?
19. If the ratio of X to Y is 4 to 3, and there are 462 Y's, how many X's are there?
20. If the ratio of games won to games played is 6 to 7 and 18 games were won, how many games were played?

E. <u>Least common multiple (LCM)</u>:

The LCM of two or more integers is used to find the lowest common denominator of fractions in order to add or subtract them.

> To find the LCM:
>
> *example:* Find the LCM of 27 and 36.
>
> First factor into primes:
>
> $27 = 3^3$
>
> $36 = 2^2 \bullet 3^2$
>
> Make the LCM by taking each prime factor to its greatest power:
>
> $\text{LCM} = 2^2 \bullet 3^3 = 4 \bullet 27 = 108$

Problems 21-25: Find the LCM:

21. 6 and 15
22. 4 and 8
23. 3 and 5
24. 8 and 12
25. 8, 12, and 15

F. <u>Lowest common denominator (LCD)</u>:

> To find LCD fractions for two or more given fractions:
>
> *example:* Given $\frac{5}{6}$ and $\frac{8}{15}$
>
> First find LCM of 6 and 15:
>
> $6 = 2 \bullet 3$
>
> $15 = 3 \bullet 5$
>
> $\text{LCM} = 2 \bullet 3 \bullet 5 = 30 = \text{LCD}$
>
> So $\frac{5}{6} = \frac{25}{30}$ and $\frac{8}{15} = \frac{16}{30}$

Problems 26-32: Find equivalent fractions with the LCD:

26. $\frac{2}{3}$ and $\frac{2}{9}$
27. $\frac{3}{8}$ and $\frac{7}{12}$
28. $\frac{4}{5}$ and $\frac{2}{3}$
29. $\frac{1}{2}, \frac{2}{3}$, and $\frac{3}{4}$
30. $\frac{7}{8}$ and $\frac{5}{8}$
31. Which is larger, $\frac{5}{7}$ or $\frac{3}{4}$? (Hint: find and compare LCD fractions)
32. Which is larger, $\frac{3}{8}$ or $\frac{1}{3}$?

G. <u>Adding and subtracting fractions</u>:

> If denominators are the same, combine the numerators:
>
> *example:* $\frac{7}{10} - \frac{1}{10} = \frac{7-1}{10} = \frac{6}{10} = \frac{3}{5}$

Problems 33-37: Find the sum or difference (reduce if possible):

33. $\frac{4}{7} + \frac{2}{7} =$
34. $\frac{5}{6} + \frac{1}{6} =$
35. $\frac{7}{8} - \frac{5}{8} =$
36. $3 + \frac{1}{2} =$
37. $1 - \frac{2}{3} =$

> If the denominators are different, first find equivalent fractions with common denominators (preferably the LCD):
>
> *example:* $\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15}$
>
> *example:* $\frac{1}{2} - \frac{2}{3} = \frac{3}{6} - \frac{4}{6} = \frac{3-4}{6} = \frac{-1}{6}$

Problems 38-43: Calculate:

38. $\frac{3}{5} - \frac{2}{3} =$
39. $\frac{5}{8} + \frac{1}{4} =$
40. $\frac{5}{2} + \frac{5}{4} =$
41. $2\frac{3}{4} + 5\frac{7}{8} =$
42. $\left(3\frac{1}{4} - \frac{3}{4}\right) + \frac{1}{2} =$
43. $4\frac{1}{3} - \left(3\frac{1}{2} - 3\right) =$

H. <u>Multiplying and dividing fractions</u>:

> To multiply fractions, multiply the tops, multiply the bottoms, and reduce if possible:
>
> *example:* $\frac{3}{4} \bullet \frac{2}{5} = \frac{3 \bullet 2}{4 \bullet 5} = \frac{6}{20} = \frac{3}{10}$

Problems 44-52: Calculate:

44. $\frac{2}{3} \bullet \frac{3}{8} =$
45. $\frac{1}{2} \bullet \frac{2}{3} =$
46. $\frac{4}{5} \times 5 =$
47. $\left(\frac{3}{4}\right)^2 =$
48. $\left(\frac{3}{2}\right)^2 =$
49. $\left(2\frac{1}{2}\right)^2 =$
50. $\frac{4}{5} \bullet 30 =$
51. $8 \bullet \frac{3}{4} =$
52. $\frac{15}{21} \times \frac{14}{25} =$

Divide fractions by making a compound fraction and then multiply the top and bottom (of the larger fraction) by the lowest common denominator (LCD) of both.

> *example:* $\frac{3}{4} \div \frac{2}{3} = \frac{\frac{3}{4}}{\frac{2}{3}}$
>
> The LCD is 12, so multiply by 12: $\frac{\frac{3}{4} \bullet 12}{\frac{2}{3} \bullet 12} = \frac{9}{8}$

example: $\dfrac{7}{\frac{2}{3}-\frac{1}{2}} = \dfrac{7\bullet 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\bullet 6}$

$= \dfrac{42}{\frac{2}{3}\bullet 6-\frac{1}{2}\bullet 6} = \dfrac{42}{4-3} = \dfrac{42}{1} = 42$

Problems 53-62: Calculate:

53. $\frac{3}{2} \div \frac{1}{4} =$
54. $11\frac{3}{8} \div \frac{3}{4} =$
55. $\frac{3}{4} \div 2 =$
56. $\dfrac{\frac{3}{4}}{\frac{2}{3}} =$
57. $\dfrac{1+\frac{1}{2}}{1-\frac{3}{4}} =$
58. $\dfrac{2}{\frac{3}{4}} =$
59. $\dfrac{\frac{2}{3}}{4} =$
60. $\frac{4}{5} \div 5 =$
61. $\frac{3}{8} \div 3 =$
62. $\dfrac{2\frac{1}{3}-\frac{1}{3}}{3\frac{2}{3}+1\frac{1}{2}} =$

I. <u>Comparing fractions</u>:

example: Arrange small to large: $\frac{7}{9}$, $\frac{5}{7}$, and $\frac{3}{4}$

LCD is $2^2 \bullet 3^2 \bullet 7 = 252$

$\frac{7}{9} = \frac{7\bullet 28}{9\bullet 28} = \frac{196}{252}$

$\frac{5}{7} = \frac{5\bullet 36}{7\bullet 36} = \frac{180}{252}$

$\frac{3}{4} = \frac{3\bullet 63}{4\bullet 63} = \frac{189}{252}$

So the order is $\frac{5}{7}$, $\frac{3}{4}$, $\frac{7}{9}$

Fractions can also be compared by writing in decimal from and comparing the decimals.

Problems 63-65: Arrange small to large:

63. $\frac{15}{8}$, $\frac{11}{6}$
64. $\frac{7}{8}$, $\frac{5}{6}$, $\frac{11}{12}$
65. $\frac{2}{3}$, $\frac{7}{12}$, $\frac{5}{6}$, $\frac{25}{36}$

<u>Word Problems</u>:

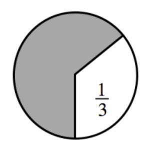
66. How many 2's are in 8?
67. How many $\frac{1}{2}$'s are in 8?
68. Three fourths is equal to how many twelfths?
69. What is $\frac{3}{4}$ of a dozen?
70. Joe and Mae are decorating the gym for a dance. Joe has done $\frac{1}{3}$ of the work and Mae has done $\frac{2}{5}$. What fraction of the work still must be done?
71. The ratio of winning tickets to tickets sold is 2 to 5. If 3,500,000 are sold, how many tickets are winners?
72. An $11\frac{3}{8}$-inch wide board can be cut into how many strips of width $\frac{5}{8}$ inch, if each cut takes $\frac{1}{8}$ inch of the width? (Must the answer be a whole number?)

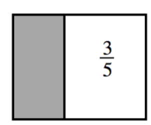
Problems 73-76: Inga and Lee each work for \$4.60 per hour:

73. If Inga works $3\frac{1}{2}$ hours, what will her pay be?
74. If Lee works $2\frac{3}{4}$ hours, what will he be paid?
75. Together, what is the total time they work?
76. What is their total pay?

<u>Visual Problems</u>:

Problems 77-80: What fraction of the figure is <u>shaded</u>?

77.


78.


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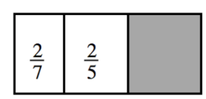
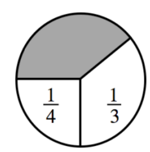
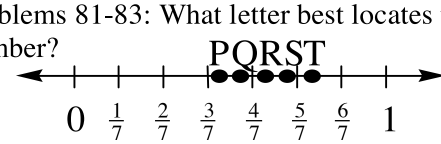


80.


Problems 81-83: What letter best locates the given number?



81. $\frac{5}{9}$
82. $\frac{3}{4}$
83. $\frac{2}{3}$

<u>Answers</u>:

1. $2^3 \bullet 3$
2. $2\bullet 3\bullet 7$
3. 1, 2, 3, 6. 9, 18
4. 1, 2, 3, 4, 6, 8, 12, 24
5. 18
6. 9
7. 1
8. $\frac{1}{4}$
9. $\frac{2}{5}$
10. $\frac{3}{4}$
11. $\frac{1}{4}$
12. $\frac{4}{7}$
13. $\frac{4}{3}$
14. 32
15. 12
16. 28
17. 6
18. 14
19. 616
20. 21
21. 30
22. 8
23. 15
24. 24
25. 120
26. $\frac{6}{9}$, $\frac{2}{9}$
27. $\frac{9}{24}$, $\frac{14}{24}$
28. $\frac{12}{15}$, $\frac{10}{15}$
29. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$
30. $\frac{7}{8}$, $\frac{5}{8}$
31. $\frac{3}{4}$ (because $\frac{20}{28} < \frac{21}{28}$)
32. $\frac{3}{8}$ (because $\frac{9}{24} > \frac{8}{24}$)

33. $\frac{6}{7}$
34. 1
35. $\frac{1}{4}$
36. $3\frac{1}{2}$
37. $\frac{1}{3}$
38. $-\frac{1}{15}$
39. $\frac{7}{8}$
40. $\frac{15}{4}$
41. $8\frac{5}{8}$
42. 3
43. $3\frac{5}{6}$
44. $\frac{1}{4}$
45. $\frac{1}{3}$
46. 4
47. $\frac{9}{16}$
48. $\frac{9}{4}$
49. $6\frac{1}{4}$ or $\frac{25}{4}$
50. 24
51. 6
52. $\frac{2}{5}$
53. 6
54. $15\frac{1}{6}$
55. $\frac{3}{8}$
56. $\frac{9}{8}$
57. 6
58. $\frac{8}{3}$
59. $\frac{1}{6}$
60. $\frac{4}{25}$
61. $\frac{1}{8}$
62. $\frac{12}{31}$
63. $\frac{11}{6}$, $\frac{15}{8}$
64. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$
65. $\frac{7}{12}$, $\frac{2}{3}$, $\frac{25}{36}$, $\frac{5}{6}$
66. 4
67. 16
68. 9
69. 9
70. $\frac{4}{15}$
71. 1,400,000
72. 18; yes
73. $16.10
74. $12.65
75. $6\frac{1}{4}$
76. $28.75
77. $\frac{2}{3}$
78. $\frac{2}{5}$
79. $\frac{11}{35}$
80. $\frac{5}{12}$
81. Q
82. T
83. S

TOPIC 3: DECIMALS

A. <u>Meaning of Places</u>:

Each digit position has a value ten times the place to its right. The part to the left of the point is the whole number part.

example: 324.519
$$= (3\times 100) + (2\times 10) + (4\times 1) + \left(5\times\tfrac{1}{10}\right) + \left(1\times\tfrac{1}{100}\right) + \left(9\times\tfrac{1}{1000}\right)$$

Problems 1-5: Which is larger?

1. .59 or .7
2. .02 or .03
3. .2 or .03
4. 1.9 or 1.09
5. .5 or .49

Problems 6-8: Arrange in order of size from smallest to largest:

6. .02, .2, .19, .0085
7. .45, .449, .451, .5
8. 4.5, 5.4, 4.49, 5.41

<u>Repeating decimals</u> are shown with a bar over the repeating block of digits:
example: $.\overline{3}$ means .333333333…
example: $.\overline{43}$ means .4343434343…
example: $.4\overline{3}$ means .4333333333…

Problems 9-10: Arrange in order, large to small:

9. $.\overline{3}$, .3, .34
10. .6, $.\overline{67}$, .67, $.6\overline{7}$, $.\overline{6}$

B. <u>Fraction-decimal conversion</u>:

Fraction to decimal: divide the top by the bottom:
example: $\frac{3}{4} = 3 \div 4 = 0.75$
example: $\frac{20}{3} = 20 \div 3 = 6.\overline{6}$
example: $3\frac{2}{5} = 3 + \frac{2}{5} = 3 + (2 \div 5)$
$= 3 + .4 = 3.4$

Problems 11-14: Write each as a decimal. If the decimal repeats, show the repeating block:

11. $\frac{5}{8} =$
12. $\frac{3}{7} =$
13. $4\frac{1}{3} =$
14. $\frac{3}{100} =$

Non-repeating decimals to fractions: say the number as a fraction, write the fraction you say; reduce if possible:
example: $.4 =$ four tenths $= \frac{4}{10} = \frac{2}{5}$
example: $3.76 =$ three and seventy six hundredths $= 3\frac{76}{100} = 3\frac{19}{25}$

Problems 15-18: Write as a fraction:

15. $.01 =$
16. $.38 =$
17. $4.9 =$
18. $1.25 =$

Comparison of fractions and decimals: usually it is easiest to convert fractions to decimals, then compare:
example: Arrange from small to large: .3, $\frac{2}{5}$, $.\overline{3}$, $\frac{2}{7}$

As decimals these are: .3, .4, .33333…, $\overline{.285714}\ldots$
So the order is: $\overline{.285714}$, .3. $.\overline{3}$, .4, or $\frac{2}{7}$, .3, $.\overline{3}$, $\frac{2}{5}$

Problems 19-21: Arrange in order, small to large:

19. $\frac{2}{3}$, .6, .67, $.\overline{67}$
20. $\frac{7}{8}$, 0.87, $\frac{13}{16}$, 0.88
21. $\frac{1}{100}$, .01, $.00\overline{9}$, $\frac{5}{500}$

Adding and subtracting decimals: like places must be combined (line up the points):
example: $4 + .3 = 4.3$

example: $3.43 + .791 + 12$:

$$\begin{array}{r} 3.430 \\ .791 \\ \underline{12.000} \\ 16.221 \end{array}$$

example: $8 - 4.96$:

$$\begin{array}{r} 8.00 \\ \underline{-4.96} \\ 3.04 \end{array}$$

example: $6.04 - (2 - 1.4) = 6.04 - .6 = 5.44$

Problems 22-30: Calculate:

22. $5.4 + .78 =$
23. $1.36 - 0.63 =$
24. $4 - .3 + .001 - .01 + .1 =$
25. $\$3.54 - \$1.68 =$
26. $\$17 - \$10.50 =$
27. $17.5 - 10 =$
28. $4 + .3 + .02 + .001 =$
29. $8.3 - 0.92 =$
30. $4.7 + 47 + 0.47 =$

C. <u>Multiplying and dividing decimals</u>:

Multiplying decimals
example: $.3 \times .5 = .15$
example: $.3 \times .2 = .06$
example: $(.03)^2 = .0009$

31. $3.24 \times 10 =$
32. $.01 \times .2 =$
33. $(.04)^2 =$
34. $5 \times 0.4 =$
35. $(.51)^2 =$

Dividing decimals: Change the problem to an equivalent whole number problem by multiplying both numbers by the same power of 10:

example: $.3 \div .03$
Multiply both by 100 to get $30 \div 3 = 10$
example: $\frac{.014}{.07}$
Multiply both by 1000 to get $14 \div 70 = .2$

36. $.013 \div 100 =$
37. $.053 \div .2 =$
38. $\frac{340}{3.4} =$
39. $\frac{8.4}{0.07} =$
40. $\frac{7.20}{2.4} =$
41. $1.44 \div 2.4 =$
42. $\frac{36.8}{10} =$

D. <u>Percent</u>:

Meaning: translate percent as hundredths:
example: 8% means 8 hundredths or .08 or $\frac{8}{100} = \frac{2}{25}$
<u>Percent-decimal conversion</u>: To change a decimal to percent form, multiply by 100 (move the point 2 places right), write the percent symbol (%):
example: $.075 = 7.5\%$
example: $1\frac{1}{4} = 1.25 = 125\%$

Problems 43-45: Write as a percent:

43. $.3 =$
44. $4 =$
45. $.085 =$

To change a percent to decimal form, move the point 2 places left (divide by 100) and drop the % symbol:
example: $8.76\% = .0876$
example: $67\% = .67$

Problems 46-49: Write as a decimal:

46. $10\% =$
47. $136\% =$
48. $.03\% =$
49. $4\% =$

<u>Solving percent problems:</u>
Step 1: Without changing the meaning, write the problems so it says "__ of __ is __", and from this, identify *a, b,* and *c*:

$\boxed{a\ \%\ \text{ of }\ b\ \text{ is }\ c}$

Problems 50-52: Write in the form *a*% of *b* is *c*, and tell the values of *a, b,* and *c*:

50. 3% of 40 is 1.2
51. 600 is 150% of 400
52. 3 out of 12 is 25%

Step 2: Given *a* and *b*, change *a*% to a decimal and multiply ("of" can be translated "multiply"). Or, given *c* and one of the others, divide *c* by the other (first change percent to decimal); if answer is *a*, write it as a percent:

example: What is 9.4% of \$5000?
Compare *a*% of *b* is *c*: 9.4% of \$5000 is __?
Given *a* and *b*: multiply:
$.094 \times \$5000 = \$470$
example: 56 problems correct out of 80 is what percent?
Compare *a*% of *b* is *c*: __% of 80 is 56?
Given *c* and other (*b*):
$56 \div 80 = .7 = 70\%$
example: 5610 people, which is 60% of the registered voters, vote in an election. How many are registered?
Compare *a*% of *b* is *c*: 60% of __ is 5610?
Given *c* and other (*a*): $5610 \div .6 = 9350$

53. 4% of 9 is what?
54. What percent of 70 is 56?
55. 15% of what is 60?
56. What is 43% of 500?
57. 10 is what percent of 40?

E. Estimation and approximation:

Rounding to one significant digit:
example: 3.67 rounds to 4
example: .0449 rounds to .04
example: 850 rounds to either 800 or 900
example: $.\overline{4} = .44444...$ rounds to .4

Problems 58-61: Round to one significant digit:

58. 45.01
59. 1.09
60. .00083
61. $0.\overline{5}$

To estimate an answer, it is often sufficient to round each given number to one significant digit, then compute:

example: $.0298 \times .000513$ Round and compute:
$.03 \times .0005 = .000015$
.000015 is the estimate

Problems 62-66: Select the best approximation of the answer:

62. $1.2346825 \times 367.003246 =$ (4, 40, 400, 4000, 40000)
63. $.0042210398 \div .01904982 =$ (.02, .2, .5, 5, 20, 50)
64. $101.7283507 + 3.14159265 =$ (2, 4, 98, 105, 400)
65. $(4.36285903)^3 =$ (12, 64, 640, 5000, 12000)
66. $1.147 - 114.7 =$ (–100, –10, 0, 10, 100)

Word Problems:

Problems 67-69: A cassette which cost $9.50 last year costs $11 now.

67. What is the amount of the increase?
68. What percent of the original price is the increase?
69. What is the percent increase?

Problems 70-71: Jodi's weekly pay is $89.20. She gets a 5% raise.

70. What will be her new weekly pay?
71. How much more will she get?

Problems 72-74: Sixty percent of those registered voted in the last election.

72. What fraction voted?
73. If there was 45,000 registered, how many voted?
74. If 33,000 voted, how many were registered?
75. A person weighs 125 pounds. Their ideal weight is 130 pounds. Their actual weight is what percent of their ideal weight?

Answers:

1. .7
2. .03
3. .2
4. 1.9
5. .5
6. .0085, .02, .19, .2
7. .449, .45, .451, .5
8. 4.49, 4.5, 5.4, 5.41
9. .34, $.\overline{3}$, .3
10. $.6\overline{7}$, $.\overline{67}$, .67, $.\overline{6}$, .6
11. .625
12. $.\overline{428571}$
13. $4.\overline{3}$
14. .03
15. $\frac{1}{100}$
16. $\frac{19}{50}$
17. $4\frac{9}{10} = \frac{49}{50}$
18. $1\frac{1}{4} = \frac{5}{4}$
19. .6, $\frac{2}{3}$, .67, $.\overline{67}$
20. $\frac{13}{16}$, .87, $\frac{7}{8}$, .88
21. all equal $\frac{1}{100}$
22. 6.18
23. .73
24. 3.791
25. $1.86
26. $6.50
27. 7.5
28. 4.321
29. 7.38
30. 52.17
31. 32.4
32. .002
33. .0016
34. 2
35. .2601
36. .00013
37. .265
38. 100
39. 120
40. 3
41. .6
42. 3.68
43. 30%
44. 400%
45. 8.5%
46. .1
47. 1.36
48. .0003
49. .04
50. 3% of 40 is 1.2; $a = 3\%$, $b = 40$, $c = 1.2$
51. 150% of 400 is 600; $a = 150\%$, $b = 400$, $c = 600$
52. 25% of 12 is 3; $a = 25\%$, $b = 12$, $c = 3$
53. .36
54. 80%
55. 400
56. 215
57. 25%
58. 50
59. 1
60. .0008
61. .6

62. 400
63. .2
64. 105
65. 64
66. –100
67. \$1.50
68. $\approx 15.8\%$
69. $\approx 15.8\%$
70. \$93.66
71. \$4.46
72. $\frac{3}{5}$
73. 27,000
74. 55,000
75. $\approx 96\%$

TOPIC 4: EXPONENTS

A. <u>Positive integer exponents</u>:

> Meaning of exponents:
> *example:* $3^4 = 3 \times 3 \times 3 \times 3$
> $= 3 \bullet 3 \bullet 3 \bullet 3 = 81$
> *example:* $4^3 = 4 \bullet 4 \bullet 4 = 64$

Problems 1-12: Find the value:

1. $3^2 =$	7. $(-2)^3 =$
2. $2^3 =$	8. $100^2 =$
3. $(-3)^2 =$	9. $(2.1)^2 =$
4. $-(3)^2 =$	10. $(-.1)^3 =$
5. $-3^2 = -\left(3^2\right) =$	11. $\left(\frac{2}{3}\right)^3 =$
6. $-2^3 =$	12. $\left(-\frac{2}{3}\right)^3 =$

> a^b means use a as a factor b times. (b is the exponent or power of a)
> *example:* 2^5 means $2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$
> 2^5 has a value 32
> 5 is the exponent or power
> 2 is the factor
> *example:* $5 \bullet 5$ can be written 5^2. Its value is 25.
> *example:* $4^1 = 4$

Problems 13-24: Write the meaning and find the value:

13. $6^3 =$	19. $(0.1)^4 =$
14. $(-4)^2 =$	20. $\left(\frac{2}{3}\right)^4 =$
15. $0^4 =$	21. $\left(1\frac{1}{2}\right)^2 =$
16. $7^1 =$	22. $2^{10} =$
17. $1^4 =$	23. $(.03)^2 =$
18. $(-1)^3 =$	24. $3^2 \bullet 2^3 =$

> *example:* $\frac{8}{2^4} = \frac{8}{16} = \frac{1}{2}$
> *example:* $\frac{6^3}{6^2} = \frac{216}{36} = 6$

Problems 25-30: Simplify:

25. $\frac{6}{3^2} =$	28. $\frac{10}{4^2 \bullet 5} =$
26. $\frac{2^5}{8} =$	29. $\frac{2^3 \bullet 2^4}{2^5 \bullet 2} =$
27. $\frac{4 \bullet 5}{10} =$	30. $\frac{5 \bullet 12}{6^2 \bullet 10} =$

Problems 31-38: Find the value:

31. $3^2 + 4^2 =$	35. $(3.1)^2 - (.03)^2 =$
32. $5^2 =$	36. $(3.1)^2 + (.03)^2 =$
33. $3^2 + 4^2 + 12^2 =$	37. $3^3 + 4^3 + 5^3 =$
34. $13^2 =$	38. $6^3 =$

B. <u>Integer exponent laws</u>:

Problems 39-40: Write the meaning (not the value):

39. $3^2 =$	40. $3^4 =$

41. Write as a power of 3: $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 =$
42. Write the meaning: $3^2 \bullet 3^4 =$
43. Write your answer to 42 as a power of 3, then find the value.
44. Now find each value and solve: $3^2 \bullet 3^4 =$
45. So $3^2 \bullet 3^4 = 3^6$. Circle each of the powers. Note how the circled numbers are related.
46. How are they related?

Problems 47-52: Write each expression as a power of the same factor:

> *example:* $3^2 \bullet 3^4 = 3^6$

47. $4^1 \bullet 4^2 =$	50. $(-1)^5 \bullet (-1)^4 =$
48. $5^3 \bullet 5^3 =$	51. $10 \bullet 10^4 =$
49. $3^3 \bullet 3 =$	52. $10 \bullet 10 =$

53. Make a formula by filling in the brackets:
$a^b \bullet a^c = a^{[\quad]}$. This is an exponent rule.

Problems 54-56: Find the value:

54. $3^6 =$	55. $3^4 =$	56. $729 \div 81 =$

> *note:* $3^6 \div 3^4 = \frac{3^6}{3^4} = \frac{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3}{3 \bullet 3 \bullet 3 \bullet 3}$
> $= \frac{3}{3} \bullet \frac{3}{3} \bullet \frac{3}{3} \bullet \frac{3}{3} \bullet 3 \bullet 3 = 1 \bullet 1 \bullet 1 \bullet 1 \bullet 3 \bullet 3 = 3^2$

57. Circle the exponents: $\frac{3^6}{3^4} = 3^2$
58. How are the circled numbers related?

Problems 59-63: Write each expression as a power:

> *example:* $\frac{3^6}{3^4} = 3^2$

59. $2^4 \div 2^4 =$	60. $\frac{2^5}{2} =$

61. $\frac{5^2}{5} =$

62. $\frac{(-4)^7}{(-4)^2} =$

63. $\frac{1^5}{1^3} =$

64. Make a formula by filling in the brackets: $\frac{a^b}{a^c} = a^{[\quad]}$. This is another exponent rule.

Problems 65-67: Find each value:

65. $4^3 =$

66. $4^6 =$

67. $\left(4^3\right)^2 = (64)^2 =$

Problems 68-69: Write the meaning of each expression:

> *example:* $\left(4^3\right)^2 = 4^3 \bullet 4^3 = 4 \bullet 4 \bullet 4 \ \bullet \ 4 \bullet 4 \bullet 4$
> $= 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 = 4^6$

68. $\left(3^2\right)^4 =$

69. $\left(5^1\right)^3 =$

70. Circle the three exponents: $\left(4^3\right)^2 = 4^6$

71. What is the relation of the circled numbers?

72. Make a rule: $\left(a^b\right)^c = a^{[\quad]}$

73. Write your three exponent rules below:

I. $a^b \bullet a^c =$

II. $\frac{a^b}{a^c} =$

III. $\left(a^b\right)^c =$

Problems 74-80: Use the rules to write each expression as a power of the factor, and tell which rule you're using:

74. $3^4 \bullet 3^6 =$

75. $\frac{2^{10}}{2^5} =$

76. $\left(2^5\right)^2 =$

77. $\left(3^4\right)^4 =$

78. $\frac{3^4}{3} =$

79. $\left(5^1\right)^2 =$

80. $10^4 \bullet 10^3 =$

C. <u>Scientific notation</u>:

Note that scientific form always looks like $a \times 10^n$, where $1 \le a < 10$, and *n* is an integer power of 10.

> *example:* $32800 = 3.2800 \times 10^4$ if the zeros in the ten's and one's places are significant. If the one's zero is not significant, write: 3.280×10^4; if neither is significant: 3.28×10^4.
> *example:* $.0040301 = 4.031 \times 10^{-3}$
> *example:* $2 \times 10^2 = 200$
> *example:* $9.9 \times 10^{-1} = .99$

Problems 81-84: Write in scientific notation:

81. 93,000,000 =

82. .000042 =

83. 5.07 =

84. $-32 =$

Problems 85-87: Write in standard notation:

85. $1.4030 \times 10^3 =$

86. $9.11 \times 10^{-2} =$

87. $4 \times 10^{-6} =$

To compute with numbers written in scientific form, separate the parts, compute, then recombine:

> *example:* $\left(3.14 \times 10^5\right)(2)$
> $= (3.14)(2) \times 10^5 = 6.28 \times 10^5$
> *example:* $\frac{4.28 \times 10^6}{2.14 \times 10^2} = \frac{4.28}{2.14} \bullet \frac{10^6}{10^2} = 2.00 \times 10^4$

Problems 88-95: Write answer in scientific notation:

88. $10^{40} \times 10^2 =$

89. $\frac{10^{40}}{10^{10}} =$

90. $\frac{1.86 \times 10^4}{3 \times 10} =$

91. $\frac{3.6 \times 10^5}{1.8 \times 10^3} =$

92. $\frac{1.8 \times 10^8}{3.6 \times 10^5} =$

93. $\left(4 \times 10^3\right)^2 =$

94. $\left(1.5 \times 10^2\right) \times \left(5 \times 10^3\right) =$

95. $\left(1.25 \times 10^2\right)\left(4 \times 10^{-2}\right) =$

D. <u>Square roots or perfect squares</u>:

$\sqrt{a} = b$ means $b^2 = a$, where $b \ge 0$. Thus $\sqrt{49} = 7$, because $7^2 = 49$. Also, $-\sqrt{49} = -7$. Note: $\sqrt{49}$ does *not* equal –7, (even though $(-7)^2$ *does* $= 49$) because –7 is not ≥ 0.

> *example:* If $\sqrt{a} = 10$, then $a = 100$, because $10^2 = a = 100$

Problems 96-99: Find the value and tell why:

96. If $\sqrt{a} = 5$ then $a =$

97. If $\sqrt{x} = 4$, then $x =$

98. If $\sqrt{36} = b$, then $b =$

99. If $\sqrt{169} = y$, then $y =$

Problems 100-110: Find the value:

100. $\sqrt{81} =$

101. $8^2 =$

102. $\sqrt{8^2} =$

103. $\sqrt{(-7)^2} =$

104. $\sqrt{6^2 + 8^2} =$

105. $\sqrt{3^2 + 4^2} =$

106. $\sqrt{3^2 + 4^2 + 12^2} =$

107. $\sqrt{17^2 - 15^2} =$

108. $\sqrt{13^2 - 12^2} =$

109. $\sqrt{4^3} =$

110. $\sqrt{3^4} =$

Answers:

1. 9
2. 8
3. 9
4. –9
5. –9
6. –8
7. –8
8. 10,000
9. 4.41
10. –.001
11. $\frac{8}{27}$
12. $-\frac{8}{27}$
13. $6 \bullet 6 \bullet 6 = 216$
14. $(-4)(-4) = 16$
15. $0 \bullet 0 \bullet 0 \bullet 0 = 0$
16. $7 = 7$
17. $1 \bullet 1 \bullet 1 \bullet 1 = 1$
18. $(-1)(-1)(-1) = -1$
19. $(.1)(.1)(.1)(.1) = .0001$
20. $\frac{2}{3} \bullet \frac{2}{3} \bullet \frac{2}{3} \bullet \frac{2}{3} = \frac{16}{81}$
21. $\frac{3}{2} \bullet \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$
22. $2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 1024$
23. $(.03)(.03) = .0009$
24. $3 \bullet 3 \bullet 2 \bullet 2 \bullet 2 = 72$
25. $\frac{2}{3}$
26. 4
27. 2
28. $\frac{1}{8}$
29. 2
30. $\frac{1}{6}$
31. 25
32. 25
33. 169
34. 169
35. 9.6091
36. 9.6109
37. 216
38. 216
39. $3 \bullet 3$
40. $3 \bullet 3 \bullet 3 \bullet 3$
41. 3^6
42. $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$
43. $3^6 = 729$
44. $9 \bullet 81 = 729$
45. $3^{(2)} \bullet 3^{(4)} = 3^{(6)}$
46. $2 + 4 = 6$
47. 4^3
48. 5^6
49. 3^4
50. $(-1)^9$
51. 10^5
52. 10^2
53. $a^b \bullet a^c = a^{[b+c]}$
54. 729
55. 81
56. 9
57. $\frac{3^{(6)}}{3^{(4)}} = 3^{(2)}$
58. $6 - 4 = 2$
59. 2^0
60. 2^4
61. 5^1
62. $(-4)^5$
63. 1^2 (or any power of 1)
64. $\frac{a^b}{a^c} = a^{[b-c]}$
65. 64
66. 4096
67. 4096
68. $3^2 \bullet 3^2 \bullet 3^2 \bullet 3^2$
69. $5 \bullet 5 \bullet 5$
70. $\left(4^{(3)}\right)^{(2)} = 4^{(6)}$
71. $3 \times 2 = 6$
72. $\left(a^b\right)^c = a^{[bc]}$
73. I. $a^b \bullet a^c = a^{b+c}$
    II. $\frac{a^b}{a^c} = a^{b-c}$
    III. $\left(a^b\right)^c = a^{bc}$
74. 3^{10}, rule I
75. 2^5, rule II
76. 2^{10}, rule III
77. 3^{16}, rule III
78. 3^3, rule II
79. 5^2, rule III
80. 10^7, rule I
81. 9.3×10^7
82. 4.2×10^{-5}
83. 5.07
84. -3.2×10
85. 1403.0
86. 0.0911
87. .000004
88. 10^{42}
89. 10^{30}
90. 6.2×10^2
91. 2.0×10^2
92. 5.0×10^2
93. 1.6×10^7
94. 7.5×10^5
95. 5
96. 25; $5^2 = 25$
97. 16; $4^2 = 16$
98. 6; $6^2 = 36$
99. 13; $13^2 = 169$
100. 9
101. 64
102. 8
103. 7
104. 10
105. 5
106. 13
107. 8
108. 5
109. 8
110. 9

TOPIC 5: EQUATIONS and EXPRESSIONS

A. Operations with literal symbols (letters):
When letters are used to represent numbers, *addition* is shown with a "plus sign" (+), and *subtraction* with a "minus sign" (–).

Multiplication is often show by writing letters together:
example: ab means *a* times *b* So do
$a \bullet b$, $a \times b$, and $(a)(b)$
example: $7 \bullet 8 = 7 \times 8 = (7)(8)$

example: $a(b+c)$ means a times $(b+c)$

Problems 1-4: What is the meaning of:

1. abc	3. c^2
2. $2a$	4. $3(a-4)$

To show *division*, fractions are often used:
example: 3 divided by 6 may be shown: $3 \div 6$ or $\frac{3}{6}$, or $6\overline{)3}$, and all have value $\frac{1}{2}$, or .5.

5. What does $\frac{4a}{3b}$ mean?

Problems 6-15: Write a fraction form and reduce to find the value (if possible):

6. $36 \div 9 =$	11. $12 \div 5y =$
7. $4 \div 36 =$	12. $6b \div 2a =$
8. $10\overline{)36} =$	13. $8r \div 10s =$
9. $1.2\overline{).06} =$	14. $a \div a =$
10. $2x \div a =$	15. $2x \div x =$

In the above exercises, notice that the fraction forms can be reduced if there is a common (shared) factor in the top and bottom:

example: $\frac{36}{9} = \frac{9\bullet 4}{9\bullet 1} = \frac{9}{9}\bullet\frac{4}{1} = 1\bullet 4 = 4$

example: $\frac{4}{36} = \frac{4\bullet 1}{4\bullet 9} = \frac{4}{4}\bullet\frac{1}{9} = 1\bullet\frac{1}{9} = \frac{1}{9}$

example: $\frac{36}{10} = \frac{2\bullet 18}{2\bullet 5} = \frac{18}{5}$ *(or* $3\frac{3}{5}$*)*

example: $\frac{6b}{2a} = \frac{2\bullet 3\bullet b}{2\bullet a} = \frac{3b}{a}$

example: $\frac{a}{a} = 1$

example: $\frac{2x}{x} = \frac{2\bullet x}{1\bullet x} = \frac{2}{1}\bullet\frac{x}{x} = \frac{2}{1}\bullet 1 = 2\bullet 1 = 2$

Problems 16-24: Reduce and simplify:

16. $\frac{3x}{3} =$	21. $\frac{abc}{3ac} =$
17. $\frac{3x}{4x} =$	22. $\frac{6x}{8xy} =$
18. $\frac{x}{2x} =$	23. $\frac{15x^2}{10x} =$
19. $\frac{12x}{3x} =$	24. $\frac{6x}{5}\bullet 5 =$
20. $\frac{12x}{3} =$	

(Hint: $\frac{6x}{5}\bullet 5 = \frac{6x}{5}\bullet\frac{5}{1} = \frac{6x\bullet 5}{5\bullet 1}$ ...)

The *distributive property* says $a(b+c) = ab + ac$. Since equality $(=)$ goes both ways, the distributive property can also be written $ab + ac = a(b+c)$. Another form it often takes is $(a+b)c = ac + bc$, or $ac + bc = (a+b)c$.

example: $3(x-y) = 3x - 3y$. Comparing this with $a(b+c) = ab + ac$, we see $a = 3$, $b = x$, and $c = -y$

example: Compare $4x + 7x = (4+7)x = 11x$ with $ac + bc = (a+b)c$; $a = 4,\ c = x,\ b = 7$
example: $4(2+3) = 4\bullet 2 + 4\bullet 3$ (The distributive property says this has value 20, whether you do $4\bullet 5$ or $8 + 12$.)
example: $4a + 6x - 2 = 2(2a + 3x - 1)$

Problems 25-35: Rewrite, using the distributive property:

25. $6(x-3) =$	27. $(3-x)2 =$
26. $4(b+2) =$	28. $4b - 8c =$

29. $4x - x =$ (Hint: think $4x - 1x$, or 4 cookies minus 1 cookie)

30. $-5(a-1) =$	33. $3a - a =$
31. $5a + 7a =$	34. $5x - x + 3x =$
32. $3a - a =$	35. $x(x+2) =$

B. <u>Evaluation of an expression by substitution</u>:

example: Find the value of $7 - 4x$, if $x = 3$:
$7 - 4x = 7 - 4\bullet 3 = 7 - 12 = -5$
example: If $a = -7$ and $b = -1$, then
$a^2b = (-7)^2(-1) = 49(-1) = -49$
example: If $x = -2$, then $3x^2 + x - 5$
$= 3(-2)^2 + (-2) - 5 = 3\bullet 4 - 2 - 5$
$= 12 - 2 - 5 = 5$
example: $8 - c = 12$ Add c to each, giving
$8 - c + c = 12 + c$, or $8 = 12 + c$.
Then subtract 12, to get $-4 = c$, or $c = -4$.
example: $\frac{a}{5} = \frac{4}{3}$ Multiply each by the lowest common denominator 15: $\frac{a}{5}\bullet 15 = \frac{4}{3}\bullet 15$, or $a\bullet 3 = 4\bullet 5$
Then divide by 3: $\frac{a\bullet 3}{3} = \frac{20}{3}$, so $a = \frac{20}{3}$

Problems 36-46: Given $x = -1, y = 3$ and $z = -3$, find the value:

36. $2x =$	42. $2x + 4y =$
37. $-z =$	43. $2x^2 - x - 1 =$
38. $xz =$	44. $(x+z)^2 =$
39. $y + z =$	45. $x^2 + z^2 =$
40. $z + 3 =$	46. $-x^2z =$
41. $y^2 + z^2 =$	

Problems 47-54: Find the value, given $a = -1,\ b = 2,\ c = 0,\ x = -3,\ y = 1$, and $z = 2$:

47. $\frac{6}{b} =$	51. $\frac{4x-3y}{3y-2x} =$
48. $\frac{x}{a} =$	52. $\frac{b}{c} =$
49. $\frac{x}{3} =$	53. $-\frac{b}{z} =$
50. $\frac{a-y}{b} =$	54. $\frac{c}{z} =$

C. <u>Solving a linear equation in one variable</u>:
Add or subtract the same thing on each side of the equation and/or multiply or divide each side by the same thing, with the goal of getting the variable alone on one side. If there are fractions, you can eliminate them by multiplying both sides of the equation by a common denominator. If the equation is a proportion, you may wish to cross-multiply.

example: $3x = 10$ Divide both sides by 3, to get $1x$: $\frac{3x}{3} = \frac{10}{3}$, or $x = \frac{10}{3}$
example: $5 + a = 3$ Subtract 5 from each side, to get $1a$ (which is *a*): $5 + a - 5 = 3 - 5$ or $a = -2$
example: $\frac{y}{3} = 12$ Multiplying both sides by 3, to get y: $\frac{y}{3} \bullet 3 = 12 \bullet 3$, which gives $y = 36$.
example: $b - 4 = 7$ Add 4, to get $1b$: $b - 4 + 4 = 7 + 4$, or $b = 11$

Problems 55-65: Solve:

55. $2x = 94$
56. $3 = \frac{6x}{5}$
57. $3x + 7 = 6$
58. $\frac{x}{3} = \frac{5}{4}$
59. $5 - x = 9$
60. $x = \frac{2x}{5} + 1$
61. $4x - 6 = x$
62. $x - 4 = \frac{x}{2} + 1$
63. $6 - 4x = x$
64. $7x - 5 = 2x + 10$
65. $4x + 5 = 3 - 2x$

Problems 66-70: Substitute the given value, then solve for the other variable:

example: If $n = r + 3$ and $r = 5$ find the value of *n*: Replacing *r* with 5 gives $n = 5 + 3 = 8$.

66. $n = r + 3,\ n = 5$
67. $n = r + 3,\ n = 1$
68. $\frac{a}{2} = b,\ b = 6$
69. $5x = y - 3,\ x = 4$
70. $5x = y - 3,\ y = 3$

D. <u>Word Problems</u>:
If an object moves at a constant rate of speed *r*, the distance *d* it travels in time *t* is given by the formula $d = rt$.

example: If $t = 5$ and $d = 50$, find *r*: Substitute the given values in $d = rt$ and solve: $50 = r \bullet 5$, giving $r = 10$.

Problems 71-72: In $d = rt$, substitute, then solve for the variable:

71. $t = 5,\ r = 50;\ d =$
72. $d = 50,\ r = 4;\ t =$

73. On a 40 mile hike, a strong walker goes 3 miles per hour. How much time will the person hike? Write an equation, then solve it.

74. "Product"
75. "Quotient"
76. "Difference"
77. "Sum"

78. The sum of two numbers is 43. One of the two numbers is 17. What is the other?
79. Write an equation which says that the sum of a number *n* and 17 is 43.
80. Write an equation which says the amount of simple interest *A* equals the product of the invested principle *P*, the rate of interest *r*, and the time *t*.
81. Use the equation of problem 80: $P = \$200,\ r = 7\%$ and $t = 5$ years. Find the amount of interest *A*.

Problems 82-83: In a rectangle which has two sides of length *a* and two sides of length *b*, the perimeter *P* is found by adding all the side lengths, or $P = 2a + 2b$.

82. If $a = 5$ and $b = 8$, find *P*.
83. If $a = 7$ and $P = 40$, find *b*.

Problems 84-85: The difference of two numbers *x* and 12 is 5.

84. If *x* is the *larger*, an equation, which says this same thing could be $x - 12 = 5$. Write an equation if x is the *smaller* of the two numbers *x* and 12.
85. Find the two possible values of *x* by solving each equation in problem 84.

Problems 86-87: Write an equation, which says:

86. *n* is 4 more than 3.
87. 4 less than *x* is 3.
88. Solve the two equations you wrote for problems 86 and 87.

<u>Answers</u>:

1. *a* times *b* times *c*
2. 2 times *a*
3. *c* times *c*
4. 3 times $(a - 4)$
5. $4a$ divided by $3b$
6. $\frac{36}{9} = 4$
7. $\frac{4}{36} = \frac{1}{9}$
8. $\frac{36}{10} = \frac{18}{5}$
9. $\frac{.06}{1.2} = \frac{6}{120} = \frac{1}{20}$
10. $\frac{2x}{a}$
11. $\frac{12}{5y}$

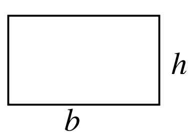
12. $\frac{6b}{2a} = \frac{3b}{a}$
13. $\frac{8r}{10s} = \frac{4r}{5s}$
14. $\frac{a}{a} = 1$
15. $\frac{2x}{x} = 2$
16. x
17. $\frac{3}{4}$
18. $\frac{1}{2}$
19. 4
20. $4x$
21. $\frac{b}{3}$
22. $\frac{3}{4y}$
23. $\frac{3x}{2}$
24. $6x$
25. $6x - 18$
26. $4b + 8$
27. $6 - 2x$
28. $4(b - 2c)$
29. $(4 - 1)x = 3x$
30. $-5a + 5$
31. $(5 + 7)a = 12a$
32. $(3 - 2)a = 1a = a$
33. $2a$
34. $7x$
35. $x^2 + 2x$
36. –2
37. 3
38. 3
39. 0
40. 0
41. 18
42. 10
43. 2
44. 16
45. 10
46. 3
47. 3
48. 3
49. –1
50. –1
51. $-\frac{5}{3}$
52. no value (undefined)
53. –1
54. 0
55. $x = 47$
56. $x = \frac{5}{2}$
57. $x = -\frac{1}{3}$
58. $x = \frac{15}{4}$
59. $x = -4$
60. $x = \frac{5}{3}$
61. $x = 2$
62. $x = 10$
63. $x = \frac{6}{5}$
64. $x = 3$
65. $x = -\frac{1}{3}$
66. $5 = r + 3;\ r = 2$
67. $1 = r + 3;\ r = -2$
68. $\frac{a}{2} = 6;\ a = 12$
69. $5 \bullet 4 = y - 3; y = 23$
70. $5x = 3 - 3; x = 0$
71. $d = 50 \bullet 5; d = 250$
72. $50 = 4t;\ t = \frac{25}{2}$
73. $40 = 3t;\ t = \frac{40}{3}$ hours
74. Multiply
75. Divide
76. Subtract
77. Add
78. 26
79. $n + 17 = 43$
80. $A = Prt$
81. $A = \$70$
82. $P = 26$
83. $b = 13$
84. $12 - x = 5$
85. $x = 17$ or 7
86. $n = 4 + 3$
87. $x - 4 = 3$
88. $n = 7;\ x = 7$

TOPIC 6: GEOMETRY

A. <u>Formulas for perimeter *P* and area *A* of rectangles, squares, parallelograms, and triangles</u>:

Rectangle with base b and altitude (height) h:

$P = 2b + 2h$
$A = bh$

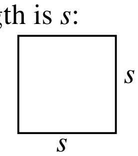


If a wire is bent in this shape, the perimeter P is the length of the wire, and the area A is the number of square units enclosed by the wire.

example: A rectangle with $b = 7$ and $h = 8$:
$P = 2b + 2h = 2 \bullet 7 + 2 \bullet 8 = 14 + 16 = 30$ units
$A = bh = 7 \bullet 8 = 56$ square units

A *square* is a rectangle with all sides equal, so the rectangle formulas apply (and simplify). If the side length is s:

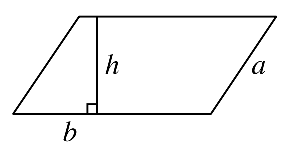
$P = 4s$
$A = s^2$



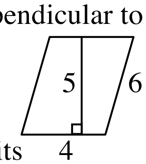
example: A square with side $s = 11$cm has
$P = 4s = 4 \times 11 = 44$ cm
$A = s^2 = 11^2 = 121\,\text{cm}^2$ (sq. cm)

A *parallelogram* with base b and height h and other side a:

$A = bh$
$P = 2a + 2b$

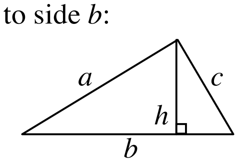


example: A parallelogram has sides 4 and 6; 5 is the length of the altitude perpendicular to the side 4.
$P = 2a + 2b = 2 \bullet 6 + 2 \bullet 4$
$= 12 + 8 = 20$ units
$A = bh = 4 \bullet 5 = 20$ square units



In a *triangle* with side lengths a, b, and c, and altitude height h to side b:

$P = a + b + c$
$A = \frac{1}{2}bh = \frac{bh}{2}$

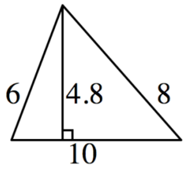


example:

$P = a + b + c$

$= 6 + 8 + 10$

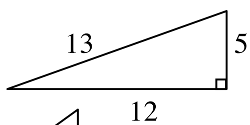
$= 24$ units



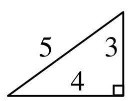
$A = \frac{1}{2}bh = \frac{1}{2}(10)(4.8) = 24$ square units

Problems 1-8: Find P and A:

1. Rectangle with sides 5 and 10.
2. Rectangle with sides 1.5 and 4.
3. Square with sides 3 miles.
4. Square with sides $\frac{3}{4}$ yards.
5. Parallelogram with sides 36 and 24, and height 10 (on side 36).
6. Parallelogram, all sides 12, altitude 6.
7. Triangle with sides 5, 12, and 13. Side 5 is the altitude on side 12.

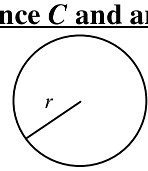


8. Triangle shown:



B. <u>Formulas for circumference C and area A of a circle</u>:

A circle with radius r (and diameter $d = 2r$) has a distance around (circumference) $C = \pi d = 2\pi r$ (If a piece of wire is bent into a circular shape, the circumference is the length of the wire.)



example: A circle with radius $r = 70$ has $d = 2r = 140$ and exact circumference $C = 2\pi r = 2 \bullet \pi \bullet 70 = 140\pi$ units

If π is approximated by $\frac{22}{7}$,

$C = 140\pi \approx 140\left(\frac{22}{7}\right) \approx 440$ units (approx.)

If π is approximated by 3.1,

$C \approx 140(3.1) = 434$ units

The area of a circle is $A = \pi r^2$

example: If $r = 8$, exact area is

$A = \pi r^2 = \pi \bullet 8^2 = 64\pi$ square units

Problems 9-11: Find the exact C and A for a circle with:

9. radius $r = 5$ units
10. $r = 10$ feet
11. diameter $d = 4$km

Problems 12-14: A circle has area 49π:

12. What is its radius length?
13. What is the diameter?
14. Find its circumference.

Problems 15-16: A parallelogram has area 48 and two sides each of length 12:

15. How long is the altitude to those sides?
16. How long are each of the other two sides?
17. How many times the P and A of a 3cm square are the P and A of a square with sides all 6 cm?
18. A rectangle has area 24 and one side 6. Find the perimeter.

Problems 19-20: A square has perimeter 30:

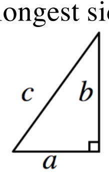
19. How long is each side?
20. What is its area?
21. A triangle has base and height each 7. What is its area?

C. <u>Pythagorean theorem</u>:

In any triangle with a $90°$ (right) angle, the sum of the squares of the legs equals the square of hypotenuse.

(The legs are the two shorter sides; the hypotenuse is the longest side.)

If the legs have lengths a and b, and c is the hypotenuse length, then $a^2 + b^2 = c^2$.



In words: " In a right triangle, leg squared plus leg squared equals hypotenuse squared."

example: A right triangle has hypotenuse 5 and one leg 3. Find the other leg. Since $leg^2 + leg^2 = hyp^2$,

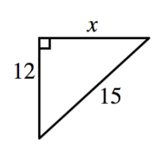
$$3^2 + x^2 = 5^2$$
$$9 + x^2 = 25$$
$$x^2 = 25 - 9 = 16$$
$$x = \sqrt{16} = 4$$

Problems 22-24: Find the length of the third side of the right triangle:

22. one leg: 15, hypotenuse: 17
23. hypotenuse: 10, one leg: 8
24. legs: 5 and 12

Problems 25-26: Find x:

25.



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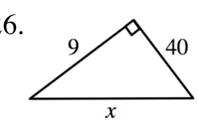

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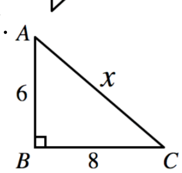
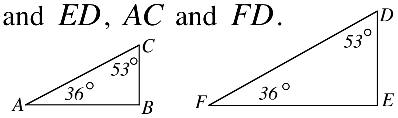


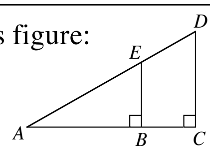
28. In right ΔRST with right angle R, $SR = 11$ and $TS = 61$. Find RT. (Draw and label a triangle to solve.)
29. Would a triangle with sides 7, 11, and 13 be a right triangle? Why or why not?

Similar triangles are triangles which are the same shape. If two angles of one triangle are equal respectively to two angles of another triangle, then the triangles are similar.

example: ΔABC and ΔFED are similar:
The pairs of sides which correspond are $\overline{AB}$ and $\overline{FE}$, $\overline{BC}$ and $\overline{ED}$, $\overline{AC}$ and $\overline{FD}$.

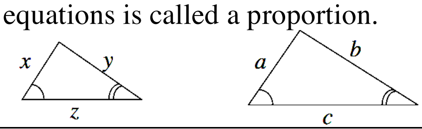


Problems 30-32: Use this figure:

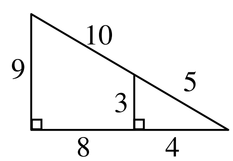


30. Find and name two similar triangles.
31. Draw the triangles separately and label them.
32. List the three pairs of corresponding sides.

If two triangles are similar, any two corresponding sides have the same ratio (fraction value):
example: the ratio a to x, or $\frac{a}{x}$, is the same as $\frac{b}{y}$ and $\frac{c}{z}$. Thus $\frac{a}{x} = \frac{b}{y}$, $\frac{a}{x} = \frac{c}{z}$, and $\frac{b}{y} = \frac{c}{z}$. Each of these equations is called a proportion.

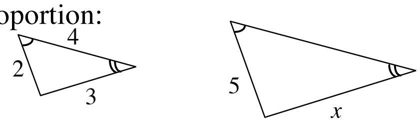


33. Draw the similar triangles separately, label them, and write proportions for the corresponding sides.



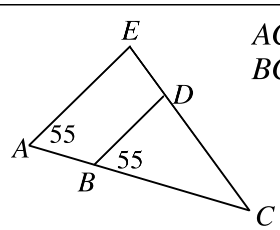
Problems 34-37: Solve for x:

example: Find x by writing and solving a proportion:

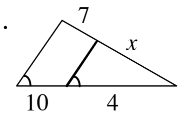


$\frac{2}{5} = \frac{3}{x}$, so cross multiply and get $2x = 15$ or $x = 7\frac{1}{2}$

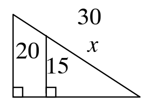
34. $AC = 10$; $EC = 7$
$BC = 4$; $DC = x$



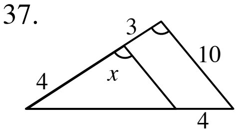
35.



36.

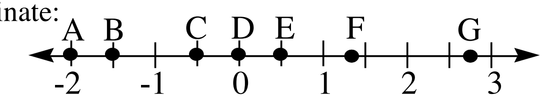


37.



D. <u>Graphing on the number line</u>:

Problems 38-45: Name the point with given coordinate:



38. 0	42. -1.5
39. $\frac{1}{2}$	43. 2.75
40. $-\frac{1}{2}$	44. $-\frac{3}{2}$
41. $\frac{4}{3}$	45. $1.\overline{3}$

Problems 46-51: On the number line above, what is the distance between the listed points? (Remember that distance is always positive.)

46. D and G	49. B and C
47. A and D	50. B and E
48. A and F	51. F and G

Problems 52-55: On the number line, find the distance from:

52. –7 to –4	54. –4 to 7
53. –7 to 4	55. 4 to 7

Problems 56-59: Draw a sketch to help find the coordinate of the point…:

56. Halfway between points with coordinates 4 and 14.
57. Midway between –5 and –1.
58. Which is the midpoint of the segment joining –8 and 4.
59. On the number line the same distance from –6 as it is from 10.

E. <u>Coordinate plane graphing</u>:

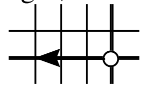
To locate a point on the plane, an ordered pair of numbers is used, written in the form (x, y).

Problems 60-63: Identify coordinates x and y in each ordered pair:

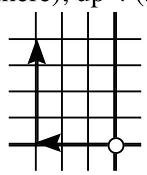
60. $(3,0)$	62. $(5,-2)$
61. $(-2,5)$	63. $(0,3)$

To plot a point, start at the origin and make the moves, first in the x-direction (horizontal) and then the y-direction (vertical) indicated by the ordered pair.

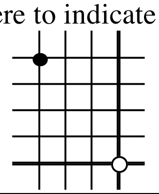
example: (-3, 4)
Start at the origin, move left 3 (since $x = -3$),



then (from there), up 4 (since $y = 4$),



put a dot there to indicate the point (-3, 4)

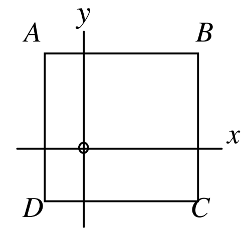


64. On graph paper, join these points in order: (–3,–2), (1,–4), (3, 0), (2, 3), (–1, 2), (3, 0), (–3, –2), (–1, 2), (1,–4).

65. Two of the lines drawn in problem 64 cross each other. What are the coordinates of the crossing point?

66. In what quadrant is the point (a,b) if $a > 0$ and $b < 0$?

Problems 67-69: $ABCD$ is a square, with $C(5,-2)$ and $D(-1,-2)$. Find:

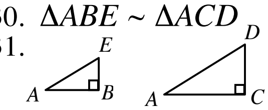


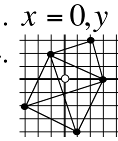
67. the length of each side.
68. the coordinates of A.
69. the coordinates of the midpoint of $\overline{DC}$.

Problems 70-72: Given $A(0,5)$, $B(12,0)$:

70. Sketch a graph. Draw $\overline{AB}$. Find its length.
71. Find the midpoint of $\overline{AB}$ and label it C. Find the coordinates of C.
72. What is the area of the triangle formed by A, B, and the origin?

Answers:

1. 30 *units*, 50 $units^2$ ($units^2$ means square units)
2. 11 *units*, 6 $units^2$
3. 12 *miles*, 9 $miles^2$
4. 3 *yards*, $\frac{9}{16}$ $yards^2$
5. 120 *un.*, 360 $un.^2$
6. 48 *un.*, 72 $un.^2$
7. 30 *un.*, 30 $un.^2$
8. 12 *un.*, 6 $un.^2$
9. 10π *un.*, 25π $un.^2$
10. 20π *ft.*, 100π $ft.^2$
11. 4π *km*, 4π km^2
12. 7
13. 14
14. 14π
15. 4
16. Cannot tell
17. P is 2 times, A is 4 times
18. 20
19. $7\frac{1}{2}$
20. $\frac{225}{4}$
21. $24\frac{1}{2}$
22. 8
23. 6
24. 13
25. 9
26. 41
27. 10
28. 60
29. No, because $7^2 + 11^2 \neq 13^2$
30. $\Delta ABE \sim \Delta ACD$
31. 

32. $\overline{AB}, \overline{AC}$; $\overline{AE}, \overline{AD}$; $\overline{BE}, \overline{CD}$
33. $\frac{3}{9} = \frac{5}{15} = \frac{4}{12}$
34. $\frac{14}{5}$ or $2\frac{4}{5}$
35. $2\frac{4}{5}$ or $\frac{14}{5}$
36. $\frac{45}{2}$
37. $\frac{40}{7}$
38. D
39. E
40. C
41. F
42. B
43. G
44. B
45. F
46. 2.75
47. 2
48. $3\frac{1}{3}$
49. 1
50. 2
51. $\frac{17}{12}$
52. 3
53. 11
54. 11
55. 3
56. 9
57. –3
58. –2
59. 2
60. $x = 3, y = 0$
61. $x = -2, y = 5$
62. $x = 5, y = -2$
63. $x = 0, y = 3$
64. 
65. (0,–1)
66. IV
67. 6
68. $(-1,4)$
69. $(2,-2)$
70. 13
71. $(6,\frac{5}{2})$
72. 30