Future Value Name:___________________________________
Math 150

Income Flows
There are 3 types of Income flows:

Type 1: Investing the same amount each time period: $f(t) = C$

Type 2: Start with an initial amount C and increase by the same amount D each time period:
$f(t) = C + Dt$

Type 3: Start with an initial amount C and increase it by the same percentage r each time period:
$f(t) = Ce^{rt}$

Examples: Write the Income flow function for each: indentify the time units for t

a) You deposit $50 per month

b) You start with $50 and increase it $5 each week

c) You start with $50 and increase it 2% each year

Future Value of an Income stream: suppose f(t) is the rate of money flow over time period T. If this income is continuously invested at a rate r compounded continuously, then the future value (money invested plus interest earned) at the end of time period T is given by:

$$Future\,Value = e^{rT} \int_0^T f(t)e^{-rt}\, dt$$

*Remember that t and T must be in the same time units, also the interest must be interest per time period.

Examples: set up the integral, *use a calculator to evaluate.*

a) What is the future value of an investment if you deposit $1000 every year for 20 years and earn 6% continuous annual interest on the investment?

b) What is the total money earned if you invest $100 a month and increase it by $10 each month for 15 years at 7% annual interest compounded continuously?

c) Suppose you start with investing $2000 a year , then increase this amount by 5% each year for 30 years. If you invest this money at 5.5% annual interest compounded continuously how much will you have at the end of the 30 years?

d) How much would you need to invest each year at 5% annual interest to have $20,000 at the end of 10 years?

Homework Problems: *use a calculator to evaluate the integrals*

1) Money is transferred continuously into an account at the constant rate of $800 per year. The account earns interest at the annual rate of 7% compounded continuously. How much will be in the account at the end of 5 years?

2) Money is transferred continuously into an account at the constant rate of $1,400 per year. Assume the account earns interest at the annual rate of 7% compounded continuously. Compute the future value of the income stream over a 18 year period.

3) If you invest $5 and increase this by $1 each week into an account that earns 8% annual interest, how much will you have at the end of 20 years?

4) Suppose you invest $500 a month and your investment earns 12% annual interest. How much will you have at the end of 10 years?

5) Money is transferred continuously into an account at the constant rate of $200 per year and you increase this by 3% each year thereafter. Assume the account earns interest at the annual rate of 6% compounded continuously. Compute the future value of the income stream over a 18 year period.

6) Money is transferred continuously into an account at the constant rate of $150 per month. Assume the account earns interest at the annual rate of 6.5% compounded continuously. Compute the future value of the income stream over a 20 year period.

7) You wish to know how much to invest each year so that after 20 years you will have $100,000 in your account. If the investment earns 10% per year, how much should you invest each year?

8) Upon the arrival of your new baby, you wish to save for college. How much do you need to invest each month, at 6% annual interest, so that in 18 years you will have $50,000 in the baby's college fund?

Math 150 Solutions to Future Value Worksheet

1) F.V. = $e^{(.07)(5)}\int_0^5 800e^{-.07t}dt$ FV= \$4789.34

2) F.V.= $e^{(.07)(18)}\int_0^{18} 1400e^{-.07t}dt$ FV = \$50,508.43

3) t= weeks (assumed 52 weeks = 1 year) F.V. = $e^{(.08/52)(1040)}\int_0^{1040}(5+1t)e^{(-.08/52)t}dt$

FV = \$1,007,003.56

4) t = months, r = .12/12=0.01 F.V. = $e^{(.01)(120)}\int_0^{120} 500e^{-.01t}dt$ FV = \$116,005.85

5) F.V. = $e^{(.06)(18)}\int_0^{18} 200e^{0.03t}e^{-0.06t}dt$ FV = \$8191.15

6) F.V. = $e^{(.065/12)(240)}\int_0^{240} 150e^{-(.065/12)t}dt$ FV=\$73,918.98

7) 100,000 = $e^{(.10)(20)}\int_0^{20} Ke^{-.1t}dt$ K = \$1565.18

They need to invest \$1565.18 each year.

8) 50,000 = $e^{(.06/12)(216)}\int_0^{216} Ke^{-(.06/12)t}dt$ K = \$128.56

They need to invest \$128.56 each month to reach their goal.