Final Exam Review
Math 150

Name:____

Solve the problem.

1) A cellular phone company determines a monthly bill from the x number of minutes of usage. The amount of the bill, B(x), (in dollars) is given by the function:

$$B(x) = 29.93 + 0.10x.$$

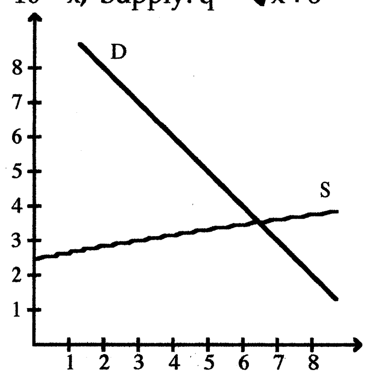
How many minutes did Marv use his phone during the month of July if his bill was $36.63?

2) A toilet manufacturer has decided to come out with a new and improved toilet. The fixed cost for the production of this new toilet line is $16,600 and the variable costs are $61 per toilet. The company expects to sell the toilets for $157. Formulate a function C(x) for the total cost of producing x new toilets and a function R(x) for the total revenue generated from the sales of x toilets.

Find the equilibrium point for the supply and demand curves. Round answers to two decimal places.

3) Demand: $q = 10 - x$, Supply: $q = \sqrt{x+6}$

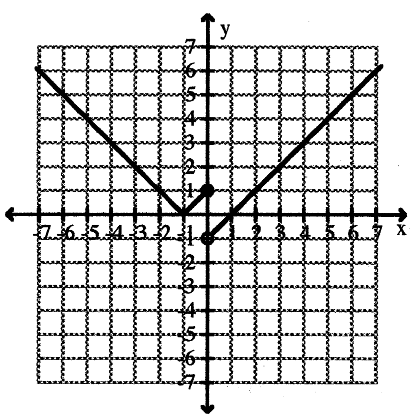
Quantity



Price

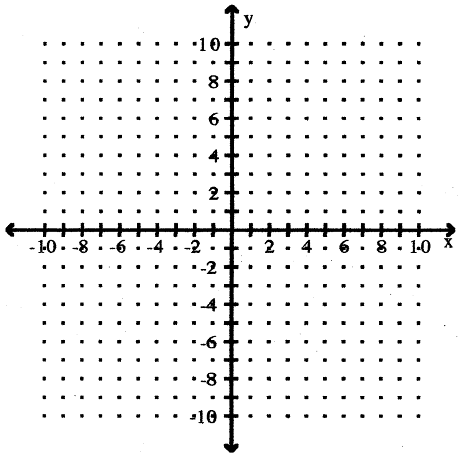
Decide whether the limit exists. If it exists, find its value.

4) Find $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.



Graph the function and then find the specified limit. When necessary, state that the limit does not exist.

5) $f(x) = \begin{cases} 2 - x, & \text{for } x \le 2, \\ 1 + 3x, & \text{for } x > 2. \end{cases}$; $\lim_{x \to 2^+} f(x)$



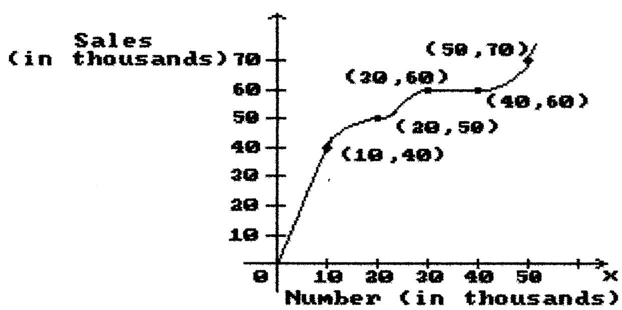
Find the limit, if it exists.

6) $\lim_{x \to 7} \sqrt{x^2 + 6x + 9}$

7) $\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$

Solve the problem.

8) The graph shows the total sales in thousands of dollars from the distribution of x thousand catalogs. Find the average rate of change of sales with respect to the number of catalogs distributed for the change in x.



10 to 40

Find the derivative of the function and evaluate the derivative at the given x–value.

9) $f(x) = x^2 + 5x$ at $x = 4$

Find an equation for the line tangent to the graph of the given function at the indicated point.

10) $f(x) = x^2 - 4$ at $(4, 12)$

Find the equation of the line tangent to the graph of the function at the indicated point.

11) $f(x) = \frac{27}{x}$ at (3, 9)

Find the derivative.

12) $y = \frac{1}{2}x^4 - \frac{1}{3}x^3$

13) $f(x) = 3x^4 - 6x^3 - 1$

14) $y = \sqrt[7]{x^6}$

15) $f(x) = \frac{4}{\sqrt{x}} - \frac{6}{x} + \frac{6}{x^4}$

Evaluate the derivative at the given value of x.

16) If $f(x) = -4x^2 + 7x - 5$, find $f'(5)$.

Find all values of x (if any) where the tangent line to the graph of the function is horizontal.

17) $y = x^3 + 5x^2 - 168x + 28$

Solve the problem.

18) If the price (in dollars) of a product is given by $P(x) = \frac{1024}{x} + 1200$, where x represents the demand for the product, find the rate of change of price when the demand is 8 units.

Find the derivative.

19) $y = (4x^2 + 4x)^2$

20) $y = \frac{x^2 - 4}{x}$

Differentiate.

21) $f(x) = \frac{5}{(2x - 3)^4}$

22) $f(x) = 4x(5x + 2)^5$

Find the equation of the line tangent to the graph of the function at the indicated point.

23) $y = (x^2 + 12)^{3/4}$ at $x = 2$

Find $\frac{d^2y}{dx^2}$.

24) $y = 6x^4 - 8x^2 + 5$

25) $y = (4x + 5)^3$

Solve the problem.

26) A population grows from an initial size of 0.7 people to an amount P(t), given by $P(t) = 0.7(7 + 0.4t + t^3)$, where t is measured in years from 1991. How rapidly is the growth rate of the population increasing t years from 1991?

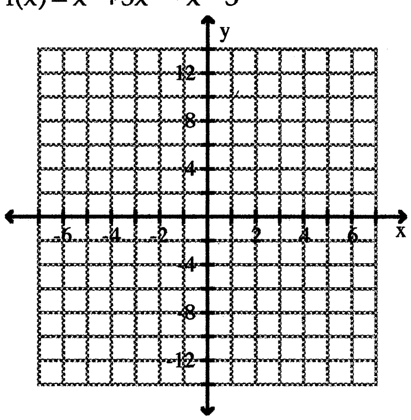
Find the relative extrema of the function, if they exist.

27) $f(x) = 2x^2 - 16x + 27$

28) $f(x) = x^4 - 18x^2 - 7$

Graph the function by first finding the relative extrema.

29) $f(x) = x^3 + 3x^2 - x - 3$



Solve the problem.

30) A firm estimates that it will sell N units of a product after spending x dollars on advertising, where $N(x) = -x^2 + 150x + 11, \quad 0 \le x \le 150$,
and x is in thousands of dollars. Find the relative extrema of the function.

Find the relative extrema of the function and classify each as a maximum or minimum.

31) $f(x) = 45x^3 - 3x^5$

Find the points of inflection.

32) $f(x) = x^3 - 3x^2 + 2x + 15$

33) $f(x) = \frac{1}{2}x^4 - 3x^3 + 12$

34) $f(x) = (x - 3)^{1/3} + 7$

Determine where the given function is increasing and where it is decreasing.

35) $f(x) = x^3 - 3x^4$

Determine where the given function is concave up and where it is concave down.

36) $f(x) = x^3 + 3x^2 - x - 24$

Solve the problem.

37) The annual revenue and cost functions for a manufacturer of grandfather clocks are approximately $R(x) = 480x - 0.02x^2$ and $C(x) = 160x + 100{,}000$, where x denotes the number of clocks made. What is the maximum annual profit?

38) Because of material shortages, it is increasingly expensive to produce 6.0L diesel engines. In fact, the profit in millions of dollars from producing x hundred thousand engines is approximated by $P(x) = -x^3 + 18x^2 + 16x - 30$, where $0 \le x \le 20$. Find the inflection point of this function to determine the point of diminishing returns.

Find the absolute maximum and absolute minimum values of the function, if they exist, over the indicated interval. When no interval is specified, use the real line $(-\infty, \infty)$.

39) $f(x) = x + \frac{16}{x}$; $[-8, -1]$

40) $f(x) = x^3 - 9x^2 + 4$; $(0, \infty)$

Solve the problem.

41) $S(x) = -x^3 + 6x^2 + 288x + 4000$, $4 \le x \le 20$ is an approximation to the number of salmon swimming upstream to spawn, where x represents the water temperature in degrees Celsius. Find the temperature that produces the maximum number of salmon. Round to the nearest tenth, if necessary.

42) The total-revenue and total-cost functions for producing x clocks are $R(x) = 480x - 0.02x^2$ and $C(x) = 200x + 100{,}000$, where $0 \le x \le 25{,}000$. What is the maximum annual profit?

43) An appliance company determines that in order to sell x dishwashers, the price per dishwasher must be
$p = 600 - 0.4x$.
It also determines that the total cost of producing x dishwashers is given by
$C(x) = 4000 + 0.6x^2$.
What is the maximum profit?

44) A hotel has 250 units. All rooms are occupied when the hotel charges $90 per day for a room. For every increase of x dollars in the daily room rate, there are x rooms vacant. Each occupied room costs $28 per day to service and maintain. What should the hotel charge per day in order to maximize daily profit?

45) An outdoor sports company sells 432 kayaks per year. It costs $12 to store one kayak for a year. Each reorder costs $8, plus an additional $7 for each kayak ordered. How many times per year should the store order kayaks in order to minimize inventory costs?

46) A supply function for a certain product is given by

$S(p) = 0.05p^3 + 4p^2 + 6p - 13$,

where S(p) is the number of items produced when the price is p dollars. Use $S'(p)$ to estimate how many more units a producer will supply when the price changes from \$13.00 per unit to \$13.20 per unit.

Find dy/dx by implicit differentiation.

47) $\frac{1}{3}x^3 - 8y^2 = 13$

48) $x^3 + 3x^2y + y^3 = 8$

For the given demand equation, differentiate implicitly to find dp/dx.

49) $9p^2 + x^2 = 1300$

Calculate dy/dt using the given information.

50) $x^{4/3} + y^{4/3} = 2;\ dx/dt = 6, x = 1, y = 1$

Solve the problem.

51) Given the revenue and cost functions $R(x) = 30x - 0.3x^2$ and $C(x) = 8x + 15$, where x is the daily production, find the rate of change of revenue with respect to time when $x = 15$ units and $\frac{dx}{dt} = 4$ units per day.

Differentiate.

52) $f(x) = e^{5x}$

53) $y = 4ex^2$

54) $f(x) = 5 - e^{-x}$

Solve the problem.

55) The sales in thousands of a new type of product are given by $S(t) = 100 - 60e^{-0.8t}$, where t represents time in years. Find the rate of change of sales at the time when $t = 3$.

Find the derivative of the function.

56) $y = \ln (x - 5)$

57) $y = \frac{\ln x}{x^6}$

Solve the problem.

58) A model for advertising response is given by

$$N(a) = 4000 + 300 \ln a, \qquad a \geq 1$$

where N(a) = the number of units sold and a = the amount spent on advertising, in thousands of dollars. Find N'(3) and Interpret

Find all relative maxima or minima.

59) $y = \ln x - x$

Solve the problem.

60) A CD store determines the following demand function for a particular CD

$$q = D(x) = \sqrt{200 - x^2},$$

where q = the number of CDs sold per day when the price per CD is x dollars. Find the elasticity at a price of $8 per CD and state whether the demand is elastic or inelastic.

61) A beverage company works out a demand function for its sale of soda and finds it to be

$$q = D(x) = 3500 - 29x,$$

where q = the quantity of sodas sold when the price per can, in cents, is x. At what price is the revenue a maximum?

Evaluate.

62) $\int 9x^7 \, dx$

63) $\int (x^3 - 5x) \, dx$

64) $\int \frac{32}{x} \, dx$

65) $\int 8e^{4x} \, dx$

Solve the problem.

66) Find a company's total-cost function if its marginal cost function is $C'(x) = 5x^2 - 7x + 4$ and $C(6) = 260$.

67) Red Plains Roasting has found that the cost, in dollars per pound, of the peanuts it roasts, is
$C'(x) = -0.015x + 7.50$, for $x \le 500$,
where x is the number of pounds of peanuts roasted. Find the total cost of roasting 280 pounds of peanuts.

68) Rejoyne Inc. has a marginal-profit function given by
$P'(x) = -2x + 140$, where $P'(x)$ is in dollars per unit.
This means that the rate of change of total profit with respect to the number of units produced, x, is $P'(x)$. Find the total profit from the production and sale of the first 40 units.

Find the area under the graph of the function over the interval given.

69) $y = 2x - x^2$; $[0, 2]$

Evaluate.

70) $\int_0^b 4e^x \, dx$

Solve the problem.

71) A manufacturer determined that its marginal cost per unit produced is given by the function

$C'(x) = 0.0006x^2 - 0.4x + 84.$

Find the total cost of producing the 301st unit through the 400th unit.

Use the numeric integration feature on your calculator to find the integral.

72) $\int_1^6 t \ln(t) \, dt$

Find the area bounded by the given curves.

73) $y = \frac{1}{2}x^2, \; y = -x^2 + 6$

Find the average value over the given interval.

74) $y = 6x + 4$; [3, 6]

Evaluate. Assume u > 0 when ln u appears.

75) $\int 8e^{4y} \, dy$

Find the consumer surplus at the equilibrium point.

76) $D(x) = 1 - 4x; x = 1$

Find the producer surplus at the equilibrium point.

77) $S(x) = \sqrt{x - 2}, 0 \le x \le 4; x = 2$

Solve the problem.

78) Find the amount of the following continuous money flow:

$R(t) = 2500t + 9, \quad k = 6\%, \quad T = 10$ years

79) Find the present value of $12,200 due 8 yr later at 11.3% compounded continuously.

Evaluate the improper integral or state that it is divergent.

80) $\int_1^\infty \frac{dx}{x^{3.325}}$

Approximate the integral using a graphing calculator

81) $\int_1^\infty \frac{4}{1 + x^2} \, dx$

Solve the problem.

82) The number of cows that can graze on a ranch is approximated by $C(x,y) = 9x + 5y - 8$, where x is the number of acres of grass and y the number of acres of alfalfa. If the ranch has 20 acres of alfalfa and 80 acres of grass, how many cows may graze?

Find the partial derivative.

83) Let $f(x, y) = 10x - 7y^2 - 5$. Find f_x.

84) Let $f(x, y) = x^3 + 4x^2y + 7xy^3$. Find f_x.

Find the second-order partial derivative.

85) Find f_{xy} when $f(x,y) = 8x^3y - 7y^2 + 2x$.

86) Find f_{yy} when $f(x,y) = 8x^3y - 7y^2 + 2x$.

Solve the problem. Assume that relative maximum and minimum values are absolute maximum and minimum values.

87) Suppose that the labor cost for a building is approximated by $C(x, y) = 10x^2 + 6y^2 - 200x - 360y + 14{,}000$, where x is the number of days of skilled labor and y is the number of days of semiskilled labor required. Find the x and y that minimize cost C.

88) A computer firm markets two kinds of electronic calculator that compete with one another. The total revenue function is $R(p, q) = 80p - 6p^2 - 4pq + 68q - 2q^2$, where p is the price of the first calculator (in multiples of \$10), and q is the price of the second calculator (in multiples of \$10). What prices should be charged in order to maximize the total revenue?

Evaluate the integral.

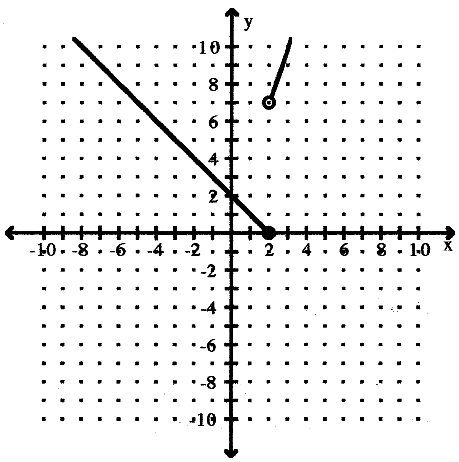
89) $\int_0^9 \int_0^5 (x + y)\, dx\, dy$

Find the volume of the solid capped by the surface z = f(x, y) over the region with the given boundaries on the xy-plane.

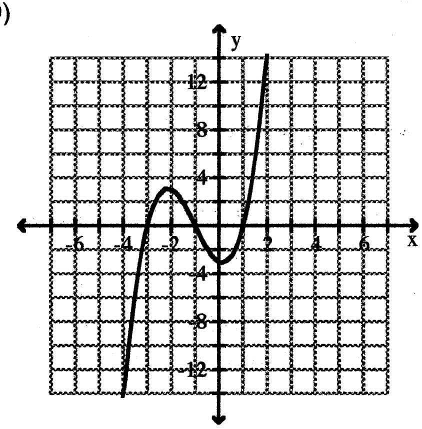
90) $z = 4x^2 + 9y^2;\ 0 \le x \le 1,\ 0 \le y \le 1$

Answer Key
Testname: MATH150FINAL REVIEW

1) 67 minutes
2) $C(x) = 16600 + 61x$; $R(x) = 157x$
3) (6.46, 3.53)
4) 1; -1
5) $\lim_{x \to 2^+} f(x) = 7$



6) 10
7) -6
8) $\frac{2}{3}$
9) $f'(x) = 2x + 5$; $f'(4) = 13$
10) $y = 8x - 20$
11) $y = -3x + 18$
12) $\frac{dy}{dx} = 2x^3 - x^2$
13) $f'(x) = 12x^3 - 18x^2$
14) $\frac{dy}{dx} = \frac{6}{7\sqrt[7]{x}}$
15) $f'(x) = -\frac{2}{x^{3/2}} + \frac{6}{x^2} - \frac{24}{x^5}$
16) -33
17) $-\frac{28}{3}$, 6
18) -\$16/unit
19) $64x^3 + 96x^2 + 32x$
20) $y' = 1 + \frac{4}{x^2}$
21) $f'(x) = \frac{-40}{(2x - 3)^5}$
22) $f'(x) = 4(5x + 2)^4(30x + 2)$
23) $y = \frac{3}{2}x + 5$
24) $72x^2 - 16$
25) $384x + 480$
26) 4.2t
27) Relative minimum at (4, -5)
28) Relative maximum at (0, -7); relative minima at (3, -88), (-3, -88)
29)



30) relative maximum at (75, 5636)
31) Relative minimum: (-3, -486), relative maximum: (3, 486)
32) (1, 15)
33) (0, 12), (3, -28.5)
34) (3, 7)
35) Increasing on $\left(-\infty, \frac{1}{4}\right]$, decreasing on $\left[\frac{1}{4}, \infty\right)$
36) Concave up on $(-1, \infty)$, concave down on $(-\infty, -1)$
37) \$1,180,000
38) (6.00, 498.00)
39) Absolute maximum: -8, absolute minimum: -17
40) No absolute maximum; absolute minimum = -104
41) 12°C
42) \$880,000
43) \$86,000
44) \$184
45) 18
46) 27
47) $\frac{x^2}{16y}$
48) $-\frac{x^2 + 2xy}{x^2 + y^2}$
49) $\frac{dp}{dx} = \frac{-x}{9p}$
50) -6

51) \$84/day
52) $5e^{5x}$
53) 8xe
54) e^{-x}
55) 4.4 thousand per year
56) $\frac{1}{x-5}$
57) $\frac{1-6\ln x}{x^7}$
58) 100
59) (1, −1), relative maximum
60) 0.47; inelastic
61) 60 cents
62) $\frac{9}{8}x^8 + C$
63) $\frac{x^4}{4} - \frac{5x^2}{2} + C$
64) $32 \ln x + C$
65) $2e^{4x} + C$
66) $C(x) = \frac{5}{3}x^3 - \frac{7}{2}x^2 + 4x + 2$
67) \$1512.00
68) \$4000
69) $\frac{4}{3}$
70) $4e^b - 4$
71) \$1782.02
72) 23.50167
73) 16
74) 31
75) $2e^{4y} + C$
76) \$2
77) \$0
78) \$188,423.65
79) \$4940.35
80) $\frac{1}{2.325}$
81) 3.1416
82) 812 cows
83) 10
84) $3x^2 + 8xy + 7y^3$
85) $24x^2$
86) −14
87) x = 10, y = 30
88) \$15 and \$155
89) 315
90) $\frac{13}{3}$