Show all your work. All answers must be simplified.

1. Find the area of the region bounded by: \( \text{(Sketch the region)} \)
   a) \( y = \cos(x) ; \quad y = 2 - \cos(x) , \quad 0 \leq x \leq 2\pi \)
   b) \( y = 2 + \sin(x) , \quad y = 1 + \cos(x) ; \quad 0 \leq x \leq \frac{3\pi}{2} \)
   c) \( x = 3y^2 - 2y , \quad x = 4y - y^2 \)

2. Find the volume of the solid generated by revolving the region bounded by \( y = 3x - x^2 \) and \( y = 0 \) about the y-axis. Include a sketch.

3. Find the volume of the solid generated by revolving the region bounded by \( 2x - y^2 = 0 \) and \( x - y = 0 \) about the y-axis. Include a sketch.

4. Find the volume of the solid generated by revolving the region bounded by \( y = x^2 \) and \( x = y^2 \) about the x-axis. Include a sketch. \( \text{(Use both washer & Shells Methods)} \)

5. \textbf{Set up}, but do not evaluate. An integral for the volume of the solid generated by revolving the region bounded by \( y = 2.5 - \cos(x) , \quad y = 0.5 , \quad x = \frac{\pi}{6} ; \quad x = \frac{5\pi}{6} , \quad \text{about the line} \quad y = -0.5 \). \( \text{(Sketch the region and a typical solid)} \)

6. \textbf{Set up}, but do not evaluate. Use the Cylindrical Shells method to set up an integral for the volume of the solid generated by revolving the region bounded by \( y = 3\sin(x) + 1 \)
   \( ; \quad y = -1 \) \( ; \quad x = 0 \) \( ; \quad x = \frac{2\pi}{3} \), about the line \( x = \frac{2\pi}{3} \). \( \text{(Sketch the region and a typical shell)} \)

7. \textbf{Set up, but do not evaluate}. Set up an integral for the volume of the solid generated by revolving the region bounded by \( y = 5 - \sec(x) , \quad y = \tan(x) - 1 , \quad x = 0 , \quad x = \frac{\pi}{3} , \quad \text{about the line} \quad y = -1 \). \( \text{(Sketch the region and a typical solid)} \)

8. A spring has a natural length of 50 cm. If a 60-N force is required to keep it stretched to a length of 72 cm, how much work is required to stretch it from 50 cm to 65 cm?
9. A hemispherical container with a radius of 6 feet is full of water. If the density of water is 62.5 lb/ft³, how much work is required to pump all the water out of the tank?

10. A tank is full of water. Find the work required to pump the water out of the outlet. Recall that water weighs 1000 kg/ m³. Assume that the ends of the tank are isosceles triangles.

11. A conical aquarium which stands point down initially has a water depth of 6 ft. If the height of the tank is 8 ft and its base radius is 4 ft, find the work required to pump the water out of the outlet. Recall that water weighs 62.5 lb/ft³.

12. \( f(x) = x^3 - 30x + 120 \); \([-1, 5]\).
   a) Find the average value of \( f \) on the given interval.
   b) Find all values of C (correct to 6 decimal places) such that \( f_{ave} = f(C) \).
   c) Sketch the graph of \( f(x) \) and a rectangle whose area is the same as the area under the graph of \( f(x) \) on \([-1, 5]\).
13. A tank is full of water. **Set up an integral** to find the work required to pump the water out of the outlet. Recall that water weighs 1000 kg/m³. Assume that the ends of the tank are isosceles trapezoids.

![Diagram of the tank](image)

14. \( f(x) = 3 \sin(\pi x) - \cos(\pi x) \) on \([-1/2, 5/3]\). 
   a) Find the average value of \( f \) on the given interval, 
   b) Find all values of \( C \) (correct to 6 decimal places) such that \( f_{ave} = f(C) \), 
   c) Sketch the graph of \( f(x) \) and a rectangle whose area is the same as the area under the graph of \( f(x) \) on \([-1/2, 5/3]\).

15. Evaluate: 
   a) \( \int \frac{4e^{\sqrt{x}}}{\sqrt{x}} \, dx \), 
   b) \( \int_{0}^{\pi} \sin(2x)\cos(x) \, dx \).

16. Integrate: 
   a) \( \int x^2 \cos(3x) \, dx \) 
   b) \( \int 2x^3 \sinh(5x) \, dx \) 
   c) \( \int e^{2x} \cosh(3x) \, dx \)
   d) \( \int \sin^{-1}(3x) \, dx \) 
   e) \( \int \tan^{-1}(4x) \, dx \) 
   f) \( \int \ln(x^2 + 4) \, dx \)
   g) \( \int 2 \sin(4\theta) \cos(7\theta) \, d\theta \) 
   h) \( \int \tan^3(x) \, dx \) 
   i) \( \int \sec^3(\theta) \tan^5(\theta) \, d\theta \)
   j) \( \int \frac{1}{x^4 \sqrt{x^2 - 9}} \, dx \) 
   k) \( \int \frac{dx}{\sqrt{x^2 + 2x + 2}} \) 
   l) \( \int \frac{dx}{\sqrt{x^2 - 4x - 5}} \)
   m) \( \int \frac{1}{\sqrt{x^2 + 36}} \, dx \) 
   n) \( \int \sqrt{x^2 - 6x + 34} \, dx \) 
   o) \( \int \sqrt{x^2 - 4x - 12} \, dx \)
   p) \( \int \frac{1 - \sin(2\theta)}{\cos(2\theta)} \, d\theta \)
   q) \( \int \sqrt{15 + 2x - x^2} \, dx \) 
   r) \( \int \frac{1}{\sqrt{24 + 2x - x^2}} \, dx \)
1. a) $4\pi \text{ unit}^2$
   b) $2 + \frac{3\pi}{2} \text{ unit}^2$
   c) $\frac{9}{4} \text{ unit}^2$

2. $\frac{27\pi}{2} \text{ unit}^3$

3. $\frac{16\pi}{\sqrt{15}} \text{ unit}^3$

4. $\frac{3\pi}{10} \text{ unit}^3$

5. \[ \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \pi \left[ 8 - 6\cos(x) + \cos'(x) \right] \, dx \text{ unit}^3 \]

6. \[ \int_{0}^{\frac{2\pi}{3}} 2\pi \left( \frac{2\pi}{3} - x \right) \left( 3\sin(x) + 2 \right) \, dx \text{ unit}^3 \]

7. \[ \int_{0}^{\frac{\pi}{3}} \pi \left[ 37 - 12\sec(x) \right] \, dx \text{ unit}^3 \]

8. $\frac{135}{44} \ J$

9. $20250 \pi \text{ ft-lb}$

10. $400,000 \ g \ J$

11. $6187.5\pi \text{ ft-lb}$

12. a) \( f_{\text{ave}} = 86 \)
   b) \( c_1 \approx 1.189423735, \ c_2 \approx 4.784781918 \in (-1, 5) \)

13. \[ \int_{0}^{5} 8000 \left[ 35 + 2y - y^2 \right] \, dx \ J \]

14. a) \( f_{\text{ave}} = \frac{3\sqrt{3 - 5}}{13\pi} \)
   b) \( c_1 \approx 0.07822995709, \ c_2 \approx 1.126602794 \in [-1/2, 5/3] \)

15. a) \( 2(e^2 - e) \)
   b) \( \frac{4}{3} \)

16. a) \( \frac{1}{27} \left[ (9x^2 - 2)\sin(3x) + 6x\cos(3x) \right] + C \)
   b) \( \frac{1}{625} \left[ (250x^3 + 60x)\cosh(5x) - (150x^2 + 12)\sinh(5x) \right] + C \)
   c) \( \frac{1}{10} \left[ e^{5x} - \frac{5}{e^x} \right] + C \)
   d) \( x\sin^{-1}(3x) + \frac{1}{3}\sqrt{1 - 9x^2} + C \)
   e) \( x\tan^{-1}(4x) - \frac{1}{8}\ln(16x^2 + 1) + C \)
   f) \( x \left[ \ln(x^2 + 4) - 2 \right] + 4\tan^{-1}\left( \frac{x}{2} \right) + C \)
   g) \( \frac{1}{33} \left[ 11\cos(3\theta) - 3\cos(11\theta) \right] + C \)
   h) \( \frac{1}{2}\sec^2(x) - \ln|\sec(x)| + C = \frac{1}{2}\tan^2(x) + \ln|\cos(x)| + C \)
   i) \( \frac{1}{2}\sec^2(x) - \frac{2}{5}\sec^5(x) + \frac{1}{3}\sec^3(x) + C \)
   j) \( \frac{(2x^2 + 9)\sqrt{x^2 - 9}}{243x^3} + C \)
   k) \( \sinh^{-1}(x + 1) + C = \ln(x + 1 + \sqrt{x^2 + 2x + 2}) + C \)
   l) \( \ln|x - 2 + \sqrt{x^2 - 2x - 5}| + C \)
   m) \( \ln|x + \sqrt{x^2 + 36}| + C \)
   n) \( \frac{1}{2} \left[ (x - 3)\sqrt{x^2 - 6x - 34} + 25\ln|x - 3 + \sqrt{x^2 - 6x - 34}| \right] + C \)
   o) \( 8\left[ (x - 2)\sqrt{x^2 - 4x - 12} - \ln|x - 2 + \sqrt{x^2 - 4x - 12}| \right] + C \)
   p) \( \frac{1}{2}\ln|1 + \sin(2\theta)| + C \)
   q) \( \frac{(x - 1)\sqrt{15 + 2x - x^2}}{2} + 8\sin^{-1}\left( \frac{x - 1}{4} \right) + C \)
   r) \( \sin^{-1}\left( \frac{x - 1}{5} \right) + C \)