

Solve each problem by the methods demonstrated in class, show your work and include graphs where indicated. Keep answers exact where possible.

1) Solve each algebraically:

a)  $2x^2 - 8x + 5 = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{8 \pm \sqrt{64 - 4(2)(5)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{24}}{4} = \frac{8}{4} \pm \frac{2\sqrt{6}}{4}$$

$x = 2 \pm \frac{\sqrt{6}}{2}$  or  $x = \frac{4 \pm \sqrt{6}}{2}$

b)  $\sqrt{2-2x} - 5 = 3x$

$$\sqrt{2-2x} = 3x+5$$

$$(\sqrt{2-2x})^2 = (3x+5)^2$$

$$2-2x = 9x^2 + 30x + 25$$

$$0 = 9x^2 + 32x + 23$$

$$0 = (9x + 23)(x + 1)$$

$x = -1$   $x = -\frac{23}{9}$

2) Solve each inequality algebraically, graph the solution set on a number line, and state the solution in interval notation:

a)  $2x^2 + 3x - 9 \geq 0$

$$(2x - 3)(x + 3) = 0$$

$$x = \frac{3}{2} \quad x = -3$$



TEST  
 $x=0$   
 NO

$(-\infty, -3] \cup [3/2, \infty)$

b)  $|\frac{1}{2}x + 5| > 2$

$$\frac{1}{2}x + 5 > 2 \quad \text{OR} \quad \frac{1}{2}x + 5 < -2$$

$$\frac{1}{2}x > -3$$

$$x > -6$$

$$\frac{1}{2}x < -7$$

$$x < -14$$



$(-\infty, -14) \cup (-6, \infty)$

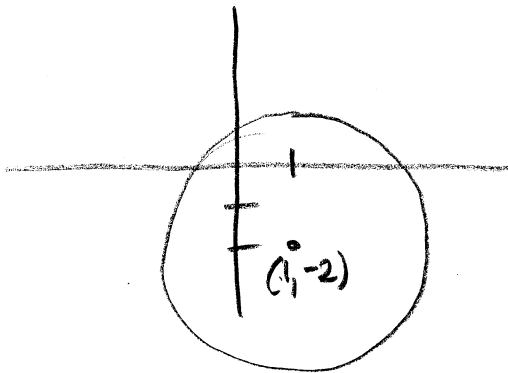
3) Find the center and radius of the circle and then sketch:  $x^2 + y^2 - 2x + 4y = 4$

$$x^2 - 2x + y^2 + 4y = 4$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$$

$$(x-1)^2 + (y+2)^2 = 9$$

$$\boxed{C(1, -2) \text{ radius} = 3}$$



4) Lines:

a) Find the equation of the line that passes through (0, -1) and (5, 9)

$$m = \frac{9 - (-1)}{5 - 0} = \frac{10}{5} = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = 2(x - 0)$$

$$y + 1 = 2x$$

$$\boxed{y = 2x - 1}$$

b) Find the equation of the line that contains (-3, -4) and is perpendicular to  $5x + 6y + 7 = 0$

use  $m = \frac{6}{5}$

$$y - y_1 = \frac{6}{5}(x - x_1)$$

$$y + 4 = \frac{6}{5}(x + 3)$$

$$y + 4 = \frac{6}{5}x + \frac{18}{5}$$

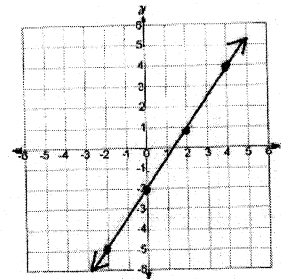
$$6y = -5x - 7$$

$$y = \left(\frac{-5}{6}\right)x - \frac{7}{6}$$

$$\rightarrow \boxed{y = \frac{6}{5}x - \frac{2}{5}}$$

c) Find the equation of the line whose graph is given:

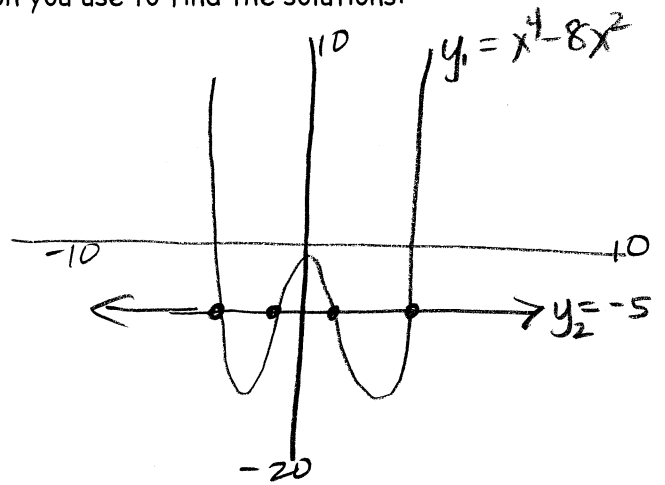
$$\boxed{y = \frac{3}{2}x - 2}$$



5) Solve graphically to two decimal places, include the graph you use to find the solutions:

$$x^4 - 8x^2 = -5$$

$$\begin{aligned} x &\approx \pm 2.70 \\ x &\approx \pm 0.83 \end{aligned}$$



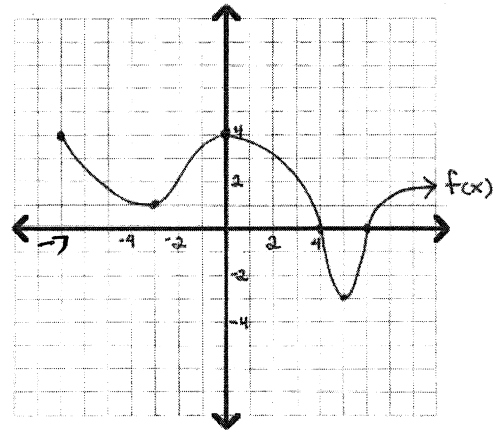
6) Given the function  $f(x)$  find each:

a) Domain:  $[-7, \infty)$

b) Range:  $[-3, 4]$

c)  $f(0) = 4$

d)  $f(-2) = 2$



e) Find all  $x$  values where  $f(x) = 4$

$$x = 0, x = -7$$

7) Given  $f(x) = 4x - 3x^2$  find each:

$$a) f(2) = 4(2) - 3(2)^2 = 8 - 12 = \boxed{-4}$$

$$b) f(-4) = 4(-4) - 3(-4)^2 = -16 - 48 = \boxed{-64}$$

$$c) f(a+1) = 4(a+1) - 3(a+1)^2$$

$$4a+4 - 3(a^2+2a+1) = \boxed{-3a^2 - 2a + 1}$$

$$4a+4 - 3a^2 - 6a - 3$$

$$d) \frac{f(a+h) - f(a)}{h} = \frac{4(a+h) - 3(a+h)^2 - (4a - 3a^2)}{h}$$

$$\frac{\cancel{4a} + 4h - 3(a^2 + 2ah + h^2) - \cancel{4a} + 3a^2}{h}$$

$$\frac{4h - 6ah - 3h^2}{h} = \boxed{4 - 6a - 3h}$$

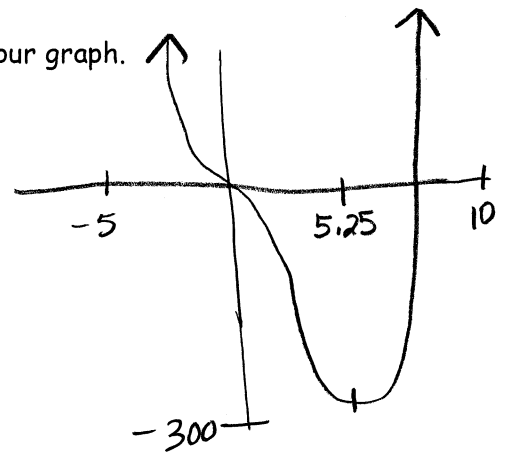
8) Graph the function  $f(x) = x^4 - 7x^3$  in an appropriate window. Show your graph.

a) State the interval(s) where is  $f(x)$  increasing

$$[5.25, \infty)$$

b) State the interval(s) where is  $f(x)$  decreasing

$$(-\infty, 5.25]$$



c) Find the average rate of change from  $x = 1$  to  $x = 3$

$$(1, -6)$$

$$(3, -108)$$

$$\frac{f(b) - f(a)}{b - a} = \frac{-108 - (-6)}{3 - 1}$$

$$\frac{-108 + 6}{2} = \frac{-102}{2} = \boxed{-51}$$

9) Given  $f(x) = x^2 - 7$ ,  $g(x) = 3x + 2$ ,  $h(x) = \frac{3 - 5x}{x + 1}$  find each:

$$\begin{aligned} \text{a) } (f \circ g)(x) &= (3x + 2)^2 - 7 \\ &= 9x^2 + 12x + 4 - 7 \\ &= \boxed{9x^2 + 12x - 3} \end{aligned}$$

$$\text{b) } (h \circ g)(x) = \frac{3 - 5(3x + 2)}{3x + 2 + 1} = \frac{3 - 15x - 10}{3x + 3} = \boxed{\frac{-15x - 7}{3x + 3}}$$

$$\begin{aligned} \text{c) } g^{-1}(x) &= \begin{aligned} x &= 3y + 2 \\ x - 2 &= 3y \\ \frac{x - 2}{3} &= y \end{aligned} \end{aligned} \quad \boxed{g^{-1}(x) = \frac{x - 2}{3}}$$

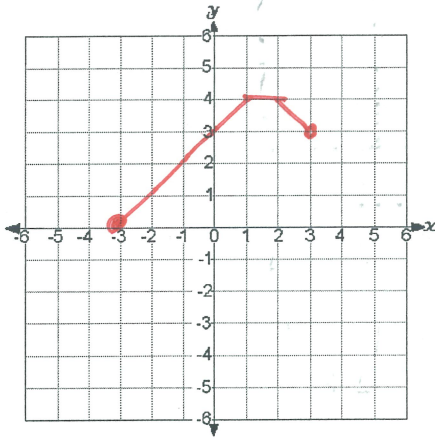
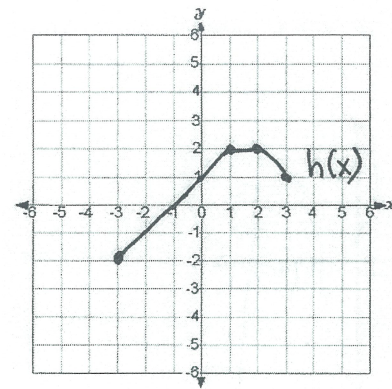
$$\begin{aligned} \text{d) } h^{-1}(x) &= \begin{aligned} x &= \frac{3 - 5y}{y + 1} \\ x(y + 1) &= 3 - 5y \\ xy + x &= 3 - 5y \\ 5y + xy &= 3 - x \\ y(5 + x) &= 3 - x \\ y &= \frac{3 - x}{5 + x} \end{aligned} \end{aligned}$$

$$\boxed{h^{-1}(x) = \frac{3 - x}{x + 5}}$$

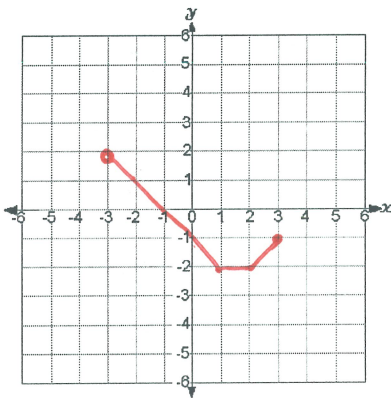
OR

$$= \frac{x - 3}{-x - 5}$$

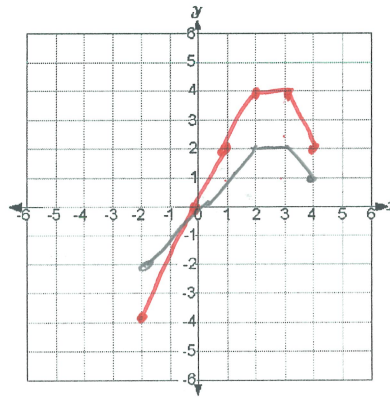
10) Given the graph  $h(x)$  at the right, sketch the graph of each transformation.



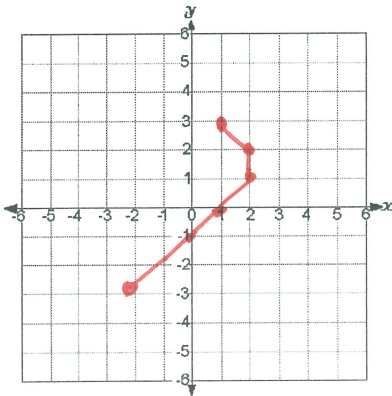
a)  $y = h(x) + 2$



b)  $y = -h(x)$



c)  $y = 2h(x - 1)$



d)  $y = h^{-1}(x)$

Extra Credit: (2 points each)

Simplify each algebraic expression, put answer in factored form where possible. No negative or rational exponents in your final answer.

a)  $10x(4x - 1)^4 + 5x^2(4x - 1)^3$

b)  $\frac{1}{2}x^{1/2}(x + 5) - 1x^{1/2}}{(x + 5)^2}$