

1) Country workshop manufactures both finished and unfinished furniture for the home. The estimated quantities demanded each week of its rolltop desks in the finished and unfinished versions are  $x$  and  $y$  units when the corresponding unit prices are:

$$p = 200 - \frac{1}{5}x - \frac{1}{10}y$$

dollars respectively.

$$q = 160 - \frac{1}{10}x - \frac{1}{4}y$$

a) Find the Revenue function  $R(x,y)$

b) Compute  $R(100, 60)$  and Interpret

2) Find the first order partial derivatives of each:

a)  $f(x,y) = x^2 - xy^2 + y^3$

b)  $f(x,y) = x \ln y + y \ln x$

c)  $f(x,y) = 3x^2 - 4xy^2 + 6y^3 + xe^{3y}$

d)  $f(x,y) = (x^2 - xy + y^2)^5$

3) Find the second order partial derivatives of  $f(x,y) = x^3 - 3x^2y + 3xy^2 + y^2$

4) The total weekly revenue of the Country workshop associated with manufacturing and selling their rolltop desks is given by:  $R(x,y) = -0.2x^2 - 0.25y^2 - 0.2xy + 200x + 160y$  where  $x$  denotes the number of finished units and  $y$  the number of unfinished units manufactured and sold each week. Compute and Interpret  $R_x(x,y)$  and  $R_y(x,y)$  when  $x = 300$  and  $y = 250$ .

5) Find any relative extrema or saddle points for:  $f(x,y) = 4y^3 + x^2 - 12y^2 - 36y + 2$

6) The total weekly revenue in dollars that Acrosonic realizes in producing and selling its bookshelf speakers is given by:  $R(x,y) = -\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 300x + 240y$  where  $x$  denotes the number of fully assembled units and  $y$  the number of unassembled kits produced and sold each week. The total weekly cost for the production of these speakers is  $C(x,y) = 180x + 140y + 5000$  dollars. Determine how many of each type of speaker units should be produced and sold each week to maximize profit. What is the maximum weekly profit?

7) Evaluate each Integral:

a)  $\int_0^4 3xy^2 dy$

b)  $\int_0^2 (3x + 5y) dx$

c)  $\int_1^2 \int_1^4 (x + 2y) dx dy$

d)  $\int_0^2 \int_x^{2x} (x^2 + y^2) dy dx$

**Answers:**

1) a. see problem 4 b. the revenue is \$25500 when selling 100 finished and 60 unfinished desks

2) a.  $f_x = 2x - y^2$  and  $f_y = -2xy + 3y^2$  b.  $f_x = Lny + \frac{y}{x}$  and  $f_y = \frac{x}{y} + Lnx$

c.  $f_x = 6x - 4y^2 + e^{3y}$  and  $f_y = -8xy + 18y^2 + 3xe^{3y}$  d.  $f_x = 5(2x - y)(x^2 - xy + y^2)^4$  and  $f_y = 5(2y - x)(x^2 - xy + y^2)^4$

3)  $f_{xx} = 6x - 6y$   $f_{yy} = 6x + 2$   $f_{xy} = f_{yx} = -6x + 6y$

4) The weekly revenue increases by \$30/unit for each additional finished desk produced (beyond 300) when the level of production of unfinished desks remains fixed at 250. Also The revenue decreases by \$25/unit when each additional unfinished desk (beyond 250) is produced and the level of production of finished desks remains fixed at 300.

5) Saddle point at (0,-1) , Relative Minimum occurs at (0,3) this relative minimum is -106

6) If they produce 208 assembled units and 64 unassembled kits the weekly profit will be maximized at \$10,680.

7) a.  $64x$  b.  $6 + 10y$  c.  $33/2$  d.  $40/3$