Mini-Lecture 3.1

Systems of Linear Equations in Two Variables

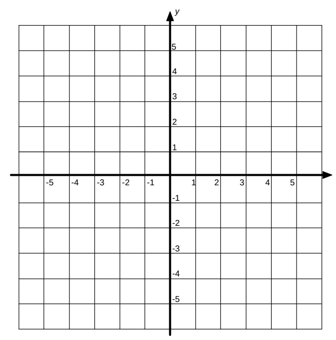
Learning Objectives:

1. Determine whether an ordered pair is a solution to a system of linear equations.
2. Solve a system of two linear equations containing two unknowns by graphing.
3. Solve a system of two linear equations containing two unknowns by substitution.
4. Solve a system of two linear equations containing two unknowns by elimination.
5. Identify inconsistent systems and dependent systems.

Preparing for Systems of Linear Equations in Two Variables:

i) Determine whether the point (–2, –2) is on the graph of $3x + 2y = -10$.

ii) Graph $8x - 3y = 12$.

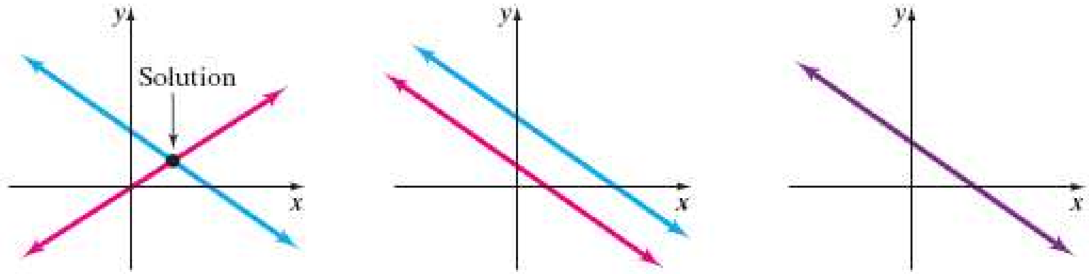


Visualizing the Solutions in a System of Two Linear Equations Containing Two Unknowns

We can view the problem of solving a system of two linear equations containing two variables as a geometry problem. The graph of each equation in the system is a line. So, a system of two equations containing two variables represents a pair of lines. The graphs of the two lines can appear in one of three ways:

1. **INTERSECT:** If the lines intersect, then the system of equations has one solution given by the point of intersection. We say that the system is **consistent** and the equations are **independent.** See Figure 1(a).
2. **PARALLEL:** If the lines are parallel, then the system of equations has no solution because the lines never intersect. In this circumstance, we say that the system is **inconsistent.** See Figure 1(b).
3. **COINCIDENT:** If the lines lie on top of each other (are coincident), then the system of equations has infinitely many solutions. The solution set is the set of all points on the line. The system is **consistent** and the equations are **dependent.** See Figure 1(c).

Figure 1



(a) Intersecting lines; system has one solution

(b) Parallel lines; system has no solution

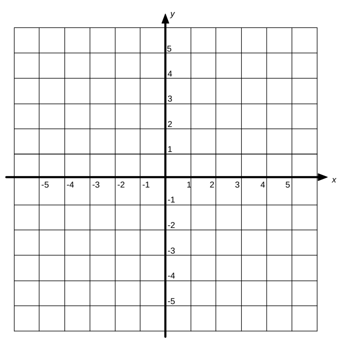
(c) Coincident lines; system has infinitely many solutions

Examples:

A **solution** of a system of equations consists of values for the variables that are solutions of each equation of the system. When we are solving systems of two linear equations containing two unknowns, we represent the solution as an ordered pair, (x, y).

1. Determine whether the ordered pair (4, –7) is a solution to the system: $\begin{cases} -x + y = -11 \\ 2x - 5y = 43 \end{cases}$.

2. Solve the system of equations by graphing: $\begin{cases} y = 2x + 5 \\ x - 3y = 0 \end{cases}$.



Steps for Solving a System of Two Linear Equations Containing Two Unknowns by Substitution

Step 1: Solve one of the equations for one of the unknowns. For example, we might solve equation (1) for y in terms of x. Choose the equation that is easiest to solve for a variable. Typically, this would be the equation that has a variable whose coefficient is 1 or -1.

Step 2: Substitute the expression that equals the variable solved for in Step 1 into the other equation. The result will be a single linear equation in one unknown. For example, if we solved equation (1) for y in terms of x in Step 1, then we would replace y in equation (2) with the algebraic expression in x.

Step 3: Solve the linear equation in one unknown found in Step 2.

Step 4: Substitute the value of the variable into the expression found in Step 1 to find the value of the other variable.

Step 5: Check your answer.

3. Solve the system of equations using substitution:

a) $\begin{cases} y = -6x + 1 \\ x = \dfrac{3}{8}y - 2 \end{cases}$

b) $\begin{cases} 3x - y = 5 \\ -2x + 5y = 1 \end{cases}$

Steps for Solving a System of Linear Equations by Elimination

Step 1: Multiply both sides of one or both equations by a nonzero constant so that the coefficients of one of the variables are additive inverses.

Step 2: Add equations (1) and (2) to eliminate the variable whose coefficients are now additive inverses. Solve the resulting equation for the unknown.

Step 3: Back-substitute the value of the variable found in Step 2 into one of the original equations to find the value of the remaining variable.

Step 4: Check your answer.

4.Solve the system of equations using elimination:

a) $\begin{cases} -x + y = 12 \\ 2x - 3y = -26 \end{cases}$ b) $\begin{cases} 3x - 5y = -1 \\ 2x - 4y = -2 \end{cases}$

5. Solve the system of equation by using either substitution or elimination.

a) $\begin{cases} y = \dfrac{4}{3}x - 2 \\ 4x - 3y = -6 \end{cases}$ b) $\begin{cases} \dfrac{1}{2}x - \dfrac{1}{5}y = \dfrac{2}{5} \\ -5x + 2y = -4 \end{cases}$