## Mini-Lecture 2.7

Variation

## Learning Objectives:

1. Model and solve problems involving direct variation.
2. Model and solve problems involving inverse variation.
3. Model and solve problems involving combined or joint variation.

## Preparing for Variation:

i) Solve: $45=9 k$
ii) Solve: $12=\frac{k}{5}$

## Examples:

## DEFINITION

Suppose we let $x$ and $y$ represent two quantities. We say that $y$ varies directly with $x$, or $y$ is directly proportional to $x$, if there is a nonzero number $k$ such that

$$
y=k x
$$

The number $k$ is called the constant of proportionality.

1. If $y$ varies directly as $x$ and $y=6$ when $x=-9$, write a linear function relating the two variables.
2. Suppose that $y$ is directly proportional to $x$ and when $x=-12, y=5$. Find $y$ when $x=20$.
3. Suppose the $C$ varies directly as $n$ and when $C=15.24, n=12$. Find $C$ when $n=37$.

## Mini-Lecture 2.7

Variation

## DEFINITION

Suppose $x$ and $y$ represent two quantities. We say that $y$ varies inversely with $x$, or $y$ is inversely proportional to $x$, if there is a nonzero number $k$ such that

$$
y=\frac{k}{x}
$$

4. Suppose that $y$ varies inversely with $x$. When $x=4, y=12$. Find $y$ when $x=18$.

> When a variable quantity $Q$ is proportional to the product of two or more other variables, we say that $Q$ varies jointly with these quantities. For example, the equation $y=k x z$ can be read as " $y$ varies jointly with $x$ and $z . "$ When direct and inverse variation occur at the same time, we have combined variation. For example, the equation $y=\frac{k x}{z}$ can be read as " $y$ varies directly with $x$ and inversely with $z . "$ The equation $y=\frac{k m n}{p}$ can be read " $y$ varies jointly with $m$ and $n$ and inversely with $p . "$
5. Suppose that $r$ varies jointly with $s$ and $t$. When $r=12, s=8$ and $t=3$. Find $r$ when $s=$ 14 and $t=6$.
6. The cost $C$ of purchasing chocolate-covered almonds varies directly with the weight $w$ in pounds. Suppose that the cost of purchasing 5 pounds of chocolate-covered almonds is $\$ 22$. What would it cost to purchase 6.5 pounds of chocolate-covered almonds?

## Mini-Lecture 2.7

Variation
7. The force $F$ of the wind on a flat surface positioned at a right angle to the direction of the wind varies jointly with the area $A$ of the surface and the square of the speed $v$ of the wind. A wind of 30 miles per hour blowing on a window measuring 3 feet by 4 feet has a force of 90 pounds. What is the force on a window measuring 6 feet by 5 feet caused by a wind of 40 miles per hour?

