Mini-Lecture 2.6

Absolute Value Equations and Inequalities

Learning Objectives:

1. Solve absolute value equations.
2. Solve absolute value inequalities involving < or ≤.
3. Solve absolute value inequalities involving > or ≥.
4. Solve applied problems involving absolute value.

Preparing for Absolute Value Equations and Inequalities:

Evaluate each expression:

i) $|-12| = 12$ *ii*) $\left|\frac{0}{-10}\right| = |0| = 0$ *iii*) $|0.4| = 0.4$

EQUATIONS INVOLVING ABSOLUTE VALUE

If a is a positive real number and if u is any algebraic expression, then

$$|u| = a \quad \text{is equivalent to} \quad u = a \quad \text{or} \quad u = -a$$

Note: If $a = 0$, the equation $|u| = 0$ is equivalent to $u = 0$. If $a < 0$, the equation $|u| = a$ has no real solution.

Examples:

1. Solve each absolute value equation.

a) $|-2x - 5| = 9$

$-2x-5=9$ or $-2x-5=-9$

$-2x=14$ $\quad -2x=-4$

$\{x \mid x=-7 \text{ or } x=2\}$

b) $3|4x+1|-2=10$

$\frac{3|4x+1|}{3} = \frac{12}{3}$

$|4x+1|=4$

$4x+1=4$ or $4x+1=-4$

$4x=3 \quad 4x=-5$

$\{x \mid x=3/4 \text{ or } x=-5/4\}$

EQUATIONS INVOLVING TWO ABSOLUTE VALUES

If u and v are any algebraic expression, then

$|u| = |v|$ is equivalent to $u = v$ or $u = -v$

c) $\left|\frac{2}{3}x - 9\right| = |6x + 3|$

$\frac{2}{3}x - 9 = 6x + 3$ or $\frac{2}{3}x - 9 = -(6x + 3)$

$2x - 27 = 18x + 9$ or $2x - 27 = -3(6x + 3)$

$-16x = 36$ $\quad$ $2x - 27 = -18x - 9$

$\frac{-16x}{-16} = \frac{36}{-16}$ $\quad$ $\frac{20x}{20} = \frac{18}{20}$

$\left\{x \mid x = -\frac{9}{4} \text{ or } x = \frac{9}{10}\right\}$

Equations with absolute values and inequalities:

If a is a positive real number and if us is an algebraic expression, then:

$|u| < a$ is equivalent to $-a < u < a$
$|u| \leq a$ is equivalent to $-a \leq u \leq a$

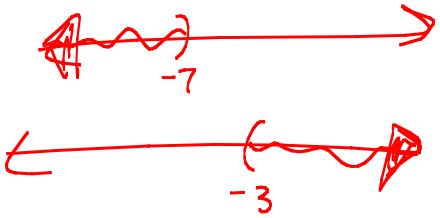
$|u| > a$ is equivalent to $u < -a$ or $u > a$
$|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$

2. Solve each absolute value inequality. Graph the solution set on a real number line.

a) $|x + 5| > 2$

$x + 5 < -2$ or $x + 5 > 2$

$x < -7$ or $x > -3$



b) $|2x - 3| \leq 7$

$2x - 3 \leq 7$ and $2x - 3 \geq -7$ $\longleftrightarrow$ $-7 \leq 2x - 3 \leq 7$

$+3 \quad +3 \quad +3$

$-\frac{4}{2} \leq \frac{2x}{2} \leq \frac{10}{2}$

$-2 \leq x \leq 5$

c) $|-5x - 8| + 12 < 4$

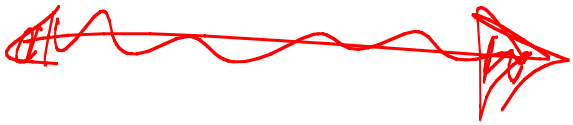
$|-5x-8| < -8$

$\emptyset$

d) $2|x + 12| \geq 0$

$\frac{2}{2} \quad \frac{}{2}$

$|x+12| \geq 0$



$(-\infty, \infty)$