Mini-Lecture 1.7

Parallel and Perpendicular Lines

Learning Objectives:

1. Define parallel lines.
2. Find equations of parallel lines.
3. Define perpendicular lines.
4. Find equations of perpendicular lines.

Preparing for Parallel and Perpendicular Lines:

i) Determine the reciprocal of $-\frac{1}{5}$.

$\boxed{-5}$

ii) Identify the slope of the line whose equation is $4x - 3y = 1$.

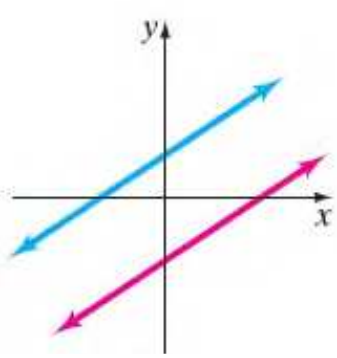
$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{1}{-3}$

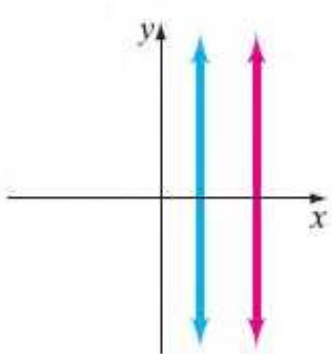
$y = \frac{4}{3}x - \frac{1}{3}$

$m = \frac{4}{3}$

DEFINITION

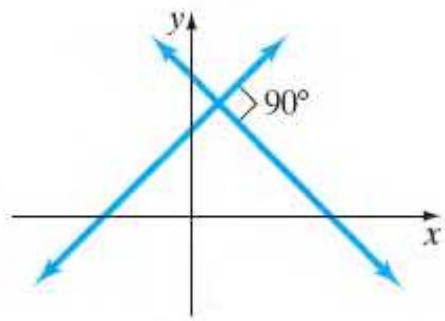
Two nonvertical lines are **parallel** if and only if their slopes are equal and they have different y-intercepts. Vertical lines are parallel if they have different x-intercepts.





DEFINITION

Two nonvertical lines are **perpendicular** if and only if the product of their slopes is -1. Alternatively, two nonvertical lines are perpendicular if their slopes are negative reciprocals of each other. Any vertical line is perpendicular to any horizontal line.



Examples:

1. Determine whether the two lines are parallel, perpendicular, or neither.

a) L_1: $2x + 3y = 9$ $\qquad$ L_2: $6x = 9y + 4$

$\boxed{\text{Neither}}$

$m = -\frac{2}{3}$ $\qquad$ $3y = -2x + 9$ $\qquad$ $y = -\frac{2}{3}x + 3$

$\frac{6x}{9} - \frac{4}{9} = \frac{9y}{9}$ $\qquad$ $m = \frac{2}{3}$

$\frac{2}{3}x - \frac{4}{9} = y$

b) L_1: $y = -\frac{7}{2}x + 3$ $\qquad$ L_2: $4x - 14y = -5$

$\boxed{\text{Perp.}}$

$m = -\frac{7}{2}$

$\frac{-14y}{-14} = \frac{-4x}{-14} - \frac{5}{-14}$ $\qquad$ $m = \frac{2}{7}$

$y = \frac{2}{7}x + \frac{5}{14}$

c) L_1: $\frac{8}{3}x - 6y = 0$ $\qquad$ L_2: $4x - 9y = 2$

$\boxed{\text{Parallel}}$

$\left(-\frac{1}{6}\right) -6y = -\frac{8}{3}x \left(-\frac{1}{6}\right)$

$m = \frac{4}{9}$ $\qquad$ $y = \frac{4}{9}x$

$-9y = -4x + 2$

$y = \frac{4}{9}x - \frac{2}{9}$ $\qquad$ $m = \frac{4}{9}$

d) L_1: contains $(-2, 3)$ and $(4, -9)$ $\qquad$ L_2: contains $(-3, -6)$ and $(1, -4)$

$\boxed{\text{Perp.}}$

$m = \frac{-9-3}{4-(-2)} = \frac{-12}{6} = -2$

$m = \frac{-4-(-6)}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$

2. Find the equation of the line with the given properties. Express the answer in slope-intercept form.

a) Parallel to $y = -3x - 2$ through the point (5, –1). $m = -3$

$y - (-1) = -3(x - 5)$

$y + 1 = -3x + 15$

$\boxed{y = -3x + 14}$

b) Parallel to $12x + 10y = 5$ through the point (–15, 0). $m = -\frac{6}{5}$

$10y = -12x + 5$

$y = \frac{-12}{10}x + \frac{5}{10}$

$y = \frac{-6}{5}x + \frac{1}{2}$

$y - 0 = \frac{-6}{5}(x - (-15))$

$\boxed{y = \frac{-6}{5}x - 18}$

c) Perpendicular to $y = -\frac{4}{5}x - 3$ through the point (10, 2). $m = \frac{5}{4}$

$y - 2 = \frac{5}{4}(x - 10)$

$y - 2 = \frac{5}{4}x - \frac{25}{2}$ $+2$ $+2$

$\boxed{y = \frac{5}{4}x - \frac{21}{2}}$

d) Perpendicular to $7x - 2y = 6$ through the point (0, –3). $m = -\frac{2}{7}$

$-2y = -7x + 6$

$y = \frac{7}{2}x - 3$

$y - (-3) = -\frac{2}{7}(x - 0)$

$y + 3 = -\frac{2}{7}x$

$\boxed{y = -\frac{2}{7}x - 3}$