Linear Equations in Two Variables

#### **Learning Objectives:**

- 1. Graph linear equations using point plotting.
- 2. Graph linear equations using intercepts.
- 3. Graph vertical and horizontal lines.
- 4. Find the slope of a line given two points.
- 5. Interpret slope as an average rate of change.
- 6. Graph a line given a point and its slope.
- 7. Use the point-slope form of a line.
- 8. Identify the slope and *y*-intercept of a line from its equation.
- 9. Find the equation of a line given two points.

#### Preparing for Linear Equations in Two Variables:

i) Solve for y: 
$$4x + 2y = -12$$
.  
 $-4x - 4x$   
 $2y = -4x -12$   
 $2 -2x -6$   
xamples:

### **Examples:**

#### DEFINITION

A linear equation in two variables is an equation of the form

$$Ax + By = C$$

where A, B, and C are real numbers. A and B cannot both be 0.

1. Graph by plotting points: x - y = 4





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# **Procedure for Finding Intercepts**

- To find the x-intercept(s), if any, of the graph of an equation, let y = 0 in the
  equation and solve for x.
- To find the y-intercept(s), if any, of the graph of an equation, let x = 0 in the
  equation and solve for y.



### DEFINITION

Let  $P = (x_1, y_1)$  and  $Q = (x_2, y_2)$  be two distinct points. If  $x_1 \neq x_2$ , the slope *m* of the nonvertical line *L* containing *P* and *Q* is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

If  $x_1 = x_2$ , then L is a vertical line and the slope m of L is **undefined** (since this results in division by 0).

4. Draw the graph of a line that contains the point (-2, -3) and has slope  $\frac{4}{3}$ .



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### POINT-SLOPE FORM OF AN EQUATION OF A LINE

An equation of a nonvertical line with slope m that contains the point  $(x_1, y_1)$  is

 $y - y_1 = \begin{array}{c} \text{Slope} \\ \downarrow \\ m(x - x_1) \\ \uparrow \\ \text{Given Point} \end{array}$ 

### SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE

An equation of a line L with slope m and y-intercept b is

$$y = f(x) = mx + b$$

5. Find the equation of the line with the given slope and containing the given point. Express your answer in slope-intercept form.

b) 
$$m = -\frac{8}{3}; (0, 2)$$
  
 $-2 = -8(x-0)$   
 $y = -\frac{5}{3}; (6, 1)$   
 $y = -\frac{5}{3}(x-6)$   
 $y = -\frac{5}{3}(x-6)$   
 $y = -\frac{5}{3}(x-6)$   
 $y = -\frac{5}{3}(x+1)$   
 $y = -\frac{5}{3}(x+1)$ 

6. Find the equation of the line containing the points (2, -2) and (-2, 6). Express your answer in slope-intercept form.

$$M = \frac{6 - (-2)}{-2 - 2} = \frac{8}{-4} = -2$$



7. Find the slope and y-intercept of: 3x + 2y - 12 = 0.

$$2y = -3x + 12 \qquad M = -\frac{3}{2} y = -\frac{3}{2}x + 6 \qquad b = 6$$

**M-10** 

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## SUMMARY: Equations of Lines

Form of Line	Formula	Comments
Horizontal Line	y = b	Graph is a horizontal line (slope is 0) with <i>y</i> -intercept <i>b</i> .
Vertical Line	x = a	Graph is a vertical line (undefined slope) with <i>x</i> -intercept <i>a</i> .
Point-slope	$y - y_1 = m(x - x_1)$	Useful for finding the equation of a line given a point and a slope or two points.
Slope-intercept	y = f(x) = mx + b	This is the form of a line expressed in function notation. Useful for quickly determining the slope and <i>y</i> -intercept of the line.
Standard	Ax + By = C	Straight forward to find the x- and y-intercepts.