## Mini-Lecture 1.6

Linear Equations in Two Variables

## Learning Objectives:

1. Graph linear equations using point plotting.
2. Graph linear equations using intercepts.
3. Graph vertical and horizontal lines.
4. Find the slope of a line given two points.
5. Interpret slope as an average rate of change.
6. Graph a line given a point and its slope.
7. Use the point-slope form of a line.
8. Identify the slope and $y$-intercept of a line from its equation.
9. Find the equation of a line given two points.

## Preparing for Linear Equations in Two Variables:

i) Solve for $y: 4 x+2 y=-12$.

$$
-4 x \quad-4 x
$$

$$
\frac{2 y}{2}=\frac{-4 x}{2} \frac{-12}{2}
$$

Examples:

$$
y=-2 x-6
$$

## DEFINITION

A linear equation in two variables is an equation of the form

$$
A x+B y=C
$$

where $A, B$, and $C$ are real numbers. $A$ and $B$ cannot both be 0 .

1. Graph by plotting points: $x-y=4$



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## Procedure for Finding Intercepts

- To find the $x$-intercept(s), if any, of the graph of an equation, let $y=0$ in the equation and solve for $x$.
- To find the $y$-intercept(s), if any, of the graph of an equation, let $x=0$ in the equation and solve for $y$.

2. Graph by finding the intercepts: $5 x+3 y=-15$

## $x$-int

$y=0$
$5 x+3(0)=15$
$5 x+3(0)=15$
$5 x=-15$ $5 x=-15$
$x=-3$ $(-3,0)$
3. Graph: $x=-2$

$m=\frac{1-0}{-2--2}=\frac{1}{0}$ undefined

## DEFINITION

Let $P=\left(x_{1}, y_{1}\right)$ and $Q=\left(x_{2}, y_{2}\right)$ be two distinct points. If $x_{1} \neq x_{2}$, the slope $\boldsymbol{m}$ of the nonvertical line $L$ containing $P$ and $Q$ is defined by the formula

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}, \quad x_{1} \neq x_{2}
$$

If $x_{1}=x_{2}$, then $L$ is a vertical line and the slope $m$ of $L$ is undefined (since this results in division by 0 ).
4. Draw the graph of a line that contains the point $(-2,-3)$ and has slope $\frac{4}{3}$ rise run


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POINT-SLOPE FORM OF AN EQUATION OF A LINE
An equation of a nonvertical line with slope $m$ that contains the point $\left(x_{1}, y_{1}\right)$ is

$$
y-y_{1}=\stackrel{\text { Slope }}{\downarrow}=m\left(x-x_{1}\right)
$$

Given Point

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE
An equation of a line $L$ with slope $m$ and $y$-intercept $b$ is

$$
y=f(x)=m x+b
$$

5. Find the equation of the line with the given slope and containing the given point. Express
your answer in slope-intercept form.
a) $m=\frac{-8}{l} ;(0,2)$

$$
\begin{aligned}
y-2 & =-8(x-0) \\
y-2 & =-8 x \\
y & =-8 x+2
\end{aligned}
$$

b) $m=-\frac{5}{3} ;(6,1)$

$$
\begin{aligned}
& y-1=-\frac{5}{3}(x-6) \\
& y-1=-\frac{5}{3} x+10 \\
& y=-\frac{5}{3} x+11
\end{aligned}
$$

6. Find the equation of the line containing the points $(2,-2)$ and $(-2,6)$. Express your answer in slope-intercept form.

$$
m=\frac{6-(-2)}{-2-2}=\frac{8}{-4}=-2
$$

$$
\begin{aligned}
& y-6=-2(x-(-2)) \\
& y-6=-2 x-4 \\
& y=-2 x+2
\end{aligned}
$$

7. Find the slope and $y$-intercept of: $3 x+2 y-12=0$.

$$
\frac{2 y}{2}=\frac{-3 x}{2}+\frac{12}{2}
$$

$$
m=-\frac{3}{2}
$$

M-10

$$
y=-\frac{3}{2} x+6
$$

$b=6$

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## SUMMARY: Equations of Lines

| Form of Line | Formula | Comments |
| :--- | :--- | :--- |
| Horizontal Line | $y=b$ | Graph is a horizontal line (slope is 0 ) with <br> $y$-intercept $b$. |
| Vertical Line | $x=a$ | Graph is a vertical line (undefined slope) with <br> $x$-intercept $a$. |
| Point-slope | $y-y_{1}=m\left(x-x_{1}\right)$ | Useful for finding the equation of a line given a <br> point and a slope or two points. |
| Slope-intercept | $y=f(x)=m x+b$ | This is the form of a line expressed in function <br> notation. Useful for quickly determining the <br> slope and $y$-intercept of the line. |
| Standard | $A x+B y=C$ | Straight forward to find the $x$ - and $y$-intercepts. |

