Mini-Lecture 1.6

Linear Equations in Two Variables

Learning Objectives:

1. Graph linear equations using point plotting.
2. Graph linear equations using intercepts.
3. Graph vertical and horizontal lines.
4. Find the slope of a line given two points.
5. Interpret slope as an average rate of change.
6. Graph a line given a point and its slope.
7. Use the point-slope form of a line.
8. Identify the slope and *y*-intercept of a line from its equation.
9. Find the equation of a line given two points.

Preparing for Linear Equations in Two Variables:

i) Solve for y: $4x + 2y = -12$.

Examples:

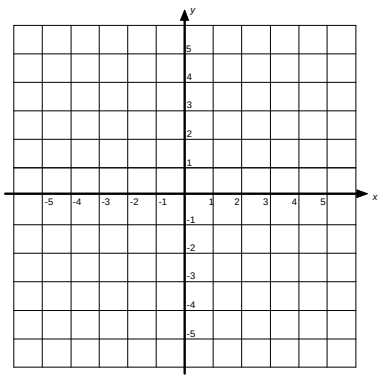
DEFINITION

A **linear equation** in two variables is an equation of the form

$$Ax + By = C$$

where A, B, and C are real numbers. A and B cannot both be 0.

1. Graph by plotting points: $x - y = 4$



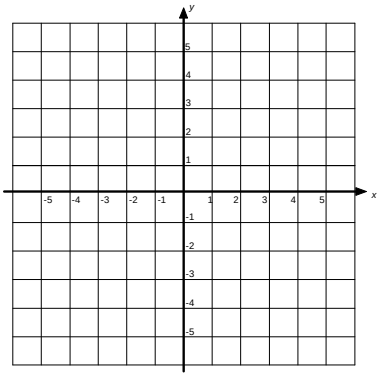
Mini-Lecture 1.6

Linear Equations in Two Variables

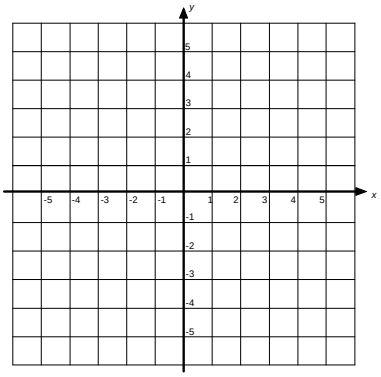
Procedure for Finding Intercepts

- To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.
- To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

2. Graph by finding the intercepts: $5x + 3y = -15$



3. Graph: $x = -2$



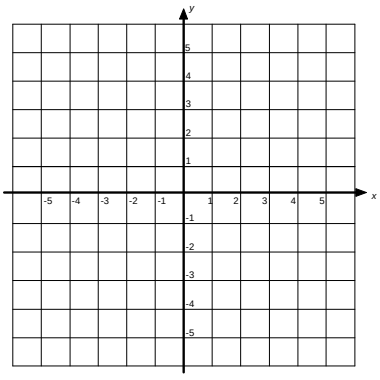
DEFINITION

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** m of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

If $x_1 = x_2$, then L is a vertical line and the slope m of L is **undefined** (since this results in division by 0).

4. Draw the graph of a line that contains the point (–2, –3) and has slope $\frac{4}{3}$.



Mini-Lecture 1.6

Linear Equations in Two Variables

POINT-SLOPE FORM OF AN EQUATION OF A LINE

An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

(Slope: m; Given Point: (x_1, y_1))

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE

An equation of a line L with slope m and y-intercept b is

$$y = f(x) = mx + b$$

5. Find the equation of the line with the given slope and containing the given point. Express your answer in slope-intercept form.

   a) $m = -8$; $(0, 2)$

   b) $m = -\frac{5}{3}$; $(6, 1)$

6. Find the equation of the line containing the points $(2, -2)$ and $(-2, 6)$. Express your answer in slope-intercept form.

7. Find the slope and y-intercept of: $3x + 2y - 12 = 0$.

Mini-Lecture 1.6

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SUMMARY: Equations of Lines

Form of Line	Formula	Comments
Horizontal Line	$y = b$	Graph is a horizontal line (slope is 0) with y-intercept b.
Vertical Line	$x = a$	Graph is a vertical line (undefined slope) with x-intercept a.
Point-slope	$y - y_1 = m(x - x_1)$	Useful for finding the equation of a line given a point and a slope or two points.
Slope-intercept	$y = f(x) = mx + b$	This is the form of a line expressed in function notation. Useful for quickly determining the slope and y-intercept of the line.
Standard	$Ax + By = C$	Straight forward to find the x- and y-intercepts.