

# Mini-Lecture 1.4

## Linear Inequalities

### Learning Objectives:

1. Represent inequalities using the real number line and interval notation.
2. Understand the properties of inequalities.
3. Solve linear inequalities.
4. Solve problems involving linear inequalities.

### Preparing for Linear Inequalities:

i) Replace the question mark by  $<$ ,  $>$ , or  $=$  to make the statement true:

$$-\frac{5 \cdot 13}{9 \cdot 13} - \frac{8 \cdot 9}{13 \cdot 9} \quad -\frac{65}{117} > -\frac{72}{117}$$

ii) Determine if the following is True or False:  $\frac{0}{-5} \leq 0$ .

$$\downarrow$$

$$0 \leq 0$$

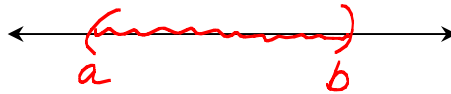
### DEFINITION: INTERVAL NOTATION

Let  $a$  and  $b$  represent two real numbers with  $a < b$ .

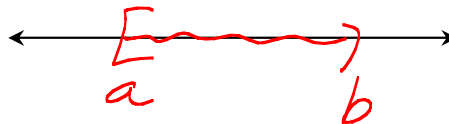
A **closed interval**, denoted by  $[a, b]$ , consists of all real numbers  $x$  for which  $a \leq x \leq b$ .



An **open interval**, denoted by  $(a, b)$ , consists of all real numbers  $x$  for which  $a < x < b$ .

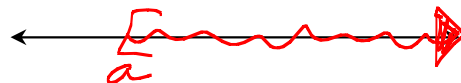


The **half-open, or half-closed, intervals** are  $(a, b]$ , consisting of all real numbers  $x$  for which  $a < x \leq b$  and  $[a, b)$ , consisting of all real numbers  $x$  for which  $a \leq x < b$ .



### INTERVALS INCLUDING $\infty$

$[a, \infty)$  consists of all real numbers  $x$  for which  $x \geq a$ .



$(a, \infty)$  consists of all real numbers  $x$  for which  $x > a$ .



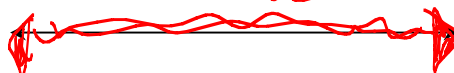
$(-\infty, a]$  consists of all real numbers  $x$  for which  $x \leq a$ .



$(-\infty, a)$  consists of all real numbers  $x$  for which  $x < a$ .



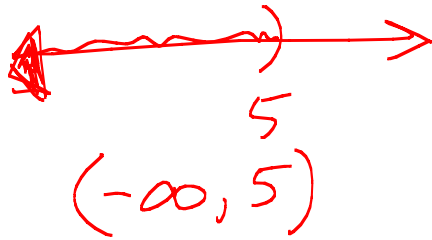
$(-\infty, \infty)$  consists of all real numbers  $x$  for which  $-\infty < x < \infty$ .



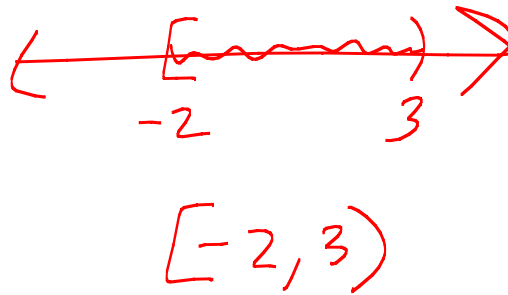
**Examples:**

1. Write in interval notation and graph the inequality.

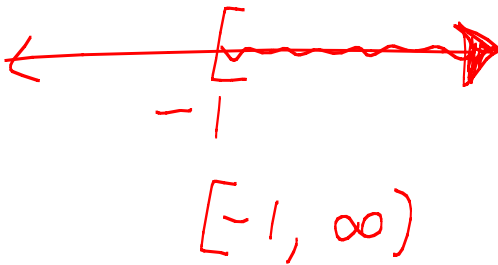
a)  $x < 5$



b)  $-2 \leq x < 3$



c)  $x \geq -1$



2. Solve each linear inequality. Express your answer in set-builder notation.

a)  $5x + 6 > -19$

$$\begin{aligned} & -6 \quad -6 \\ 5x & > -25 \\ \frac{5x}{5} & \frac{-25}{5} \\ x & > -5 \end{aligned}$$

$$\{x \mid x > -5\}$$

b)  $\frac{-3}{2} - \frac{2}{3}x \geq 4 \quad \cdot \frac{-3}{2}$

$$x \leq -6$$

$$\{x \mid x \leq -6\}$$

3. Solve each linear inequality. Express your answer in interval notation.

a)  $-2x - 5 \leq 7x - 23$

$$\begin{array}{r} -7x \\ -7x \end{array}$$

$$\begin{array}{r} -9x - 5 \leq -23 \\ +5 \quad +5 \end{array}$$

$$-9x \leq -18$$

$$\begin{array}{l} \swarrow \\ -\frac{9x}{-9} \geq \frac{-18}{-9} \rightarrow x \geq 2 \end{array}$$

$$[2, \infty)$$

b)  $3(3x + 4) < -3(x - 6)$

$$9x + 12 < -3x + 18$$

$$12x + 12 < 18$$

$$\frac{12x}{12} < \frac{6}{12}$$

$$x < \frac{1}{2}$$

$$(-\infty, \frac{1}{2})$$

4. Solve the linear inequality and graph the solution set:  $\frac{x}{2} > \frac{3}{2} - \frac{3x-1}{5}$

$$2x > 9 - (3x - 1)$$

$$2x > 9 - 3x + 1$$

$$2x > 10 - 3x$$

$$\frac{5x}{5} > \frac{10}{5}$$

$$x > 2$$



5. **Commissions** Nghiep sells digital cameras. His annual salary is \$25,000 plus a commission of 5% on all of his sales. What is the value of the digital cameras Nghiep needs to sell so that his annual salary will be at least \$36,000?

$x = \text{sales}$

$$25,000 + .05x \geq 36,000$$

$$.05x \geq 11,000$$

$$x \geq 220,000$$

Nghiep needs to sell  
at least \$220,000

