# Mini-Lecture 1.1

Linear Equations

1. Determine whether a number is a solution to an equation.
2. Solve linear equations.
3. Determine whether an equation is a conditional equation, identity, or contradiction.

**<u>Preparing for Linear Equations:</u>**

*i*) Find the LCD of $\frac{5 \cdot 2}{6 \cdot 2}$ and $\frac{3 \cdot 3}{4 \cdot 3}$.

$\frac{10}{12}$ $\frac{9}{12}$

*ii*) Simplify: $-3(2x - 5) - 12 - 8x$

$-6x + 15 - 12 - 8x$

$-14x + 3$

**<u>Examples</u>:**

1. Determine which of the numbers are solutions to: $4t - 8 = 7(t + 1)$.

a) $t = 5$ ✗

b) $t = 3$ ✗

$4t - 8 = 7t + 7$

$-7t \quad -7t$

$-3t - 8 = 7$

$+8 \quad +8$

$\frac{-3t}{-3} = \frac{15}{-3}$

$t = -5$

c) $t = -15$ ✗

d) $t = -5$

$4(-5) - 8 = 7(-5+1)$

$-20 - 8 = 7(-4)$

$-28 = -28$

**STEPS FOR SOLVING A LINEAR EQUATION**

**Step 1:** Remove any parentheses using the Distributive Property.

**Step 2:** Combine like terms on each side of the equation.

**Step 3:** Use the Addition Property of Equality to get all variables on one side of the equation and all constants on the other side.

**Step 4:** Use the Multiplication Property of Equality to get the coefficient of the variable to equal .

**Step 5:** Check your answer to be sure that it satisfies the original equation.

**ADDITION PROPERTY OF EQUALITY**

The **Addition Property of Equality** states that for real numbers $a$, $b$, and $c$,

$$\text{if } a = b, \quad \text{then} \quad a + c = b + c$$

**MULTIPLICATION PROPERTY OF EQUALITY**

The **Multiplication Property of Equality** states that for real numbers $a$, $b$, and $c$ where $c \neq 0$,

$$\text{if } a = b, \quad \text{then} \quad ac = bc$$

2.

2. Solve each linear equation. Be sure to verify your solution.

a) $6x - 5 = -23$

$+5 \quad +5$

$\frac{6x}{6} = \frac{-18}{6}$

$\boxed{x = -3}$

b) $-3a + 10 = 8a - 12$

$-8a \quad -8a$

$-11a + 10 = -12$

$-10 \quad -10$

$\frac{-11a}{-11} = \frac{-22}{-11}$

$a = 2$

c) $7 - 3w + 9w = 10 + 9w - 12$

$7 + 6w = -2 + 9w$

$+2 \quad -6w \quad +2 \quad -6w$

$\frac{9}{3} = \frac{3w}{3}$

$3 = w$

d) $4(x + 3) - 2(3x - 1) = 6 + 4(x - 2)$

$4x + 12 - 6x + 2 = 6 + 4x - 8$

$-2x + 14 = -2 + 4x$

$+2x \quad +2 \quad +2 \quad +2x$

$\frac{16}{6} = \frac{6x}{6}$

$x = \frac{16}{6}$

$x = \frac{8}{3}$

e) $\frac{2x + 3}{4} \cdot 3 + \frac{x - 1}{3} \cdot 4 = \frac{5x}{2} \cdot 6$

LCD = 12

$\frac{(6x + 9 + 4x - 4) \cdot \cancel{12}}{\cancel{12}} = \frac{(30x) \cdot \cancel{12}}{\cancel{12}}$

$6x + 9 + 4x - 4 = 30x$

$10x + 5 = 30x$

f) $\left(\frac{3p + 1}{8} - 2 = \frac{p}{6}\right) 24$

$9p + 3 - 48 = 4p$

$\frac{5p}{5} = \frac{45}{5}$

$p = 9$

$-10x \qquad -10x$

$\frac{5}{20} = \frac{20x}{20}$

$x = \frac{1}{4}$

LCD = 18 g) $\frac{2x}{3} - \frac{x+1}{9} = \frac{3x-2}{6}$

$18 \cdot \frac{2x}{3} - 18 \cdot \frac{x+1}{9} = 18 \cdot \frac{3x-2}{6}$

$12x - (2x+2) = 9x - 6$

$12x - 2x - 2 = 9x - 6$

$10x - 2 = 9x - 6$

$\boxed{x = -4}$

h) $10[0.2(3-x) = -0.4(2x-4)]10$

$2(3-x) = -4(2x-4)$

$6 - 2x = -8x + 16$

$\frac{6x}{6} = \frac{16}{6}$

$x = 5/3$

**DEFINITIONS**

A **conditional equation** is an equation that is true for some values of the variable and false for other values of the variable.

An equation that is satisfied for every choice of the variable for which both sides of the equation are defined is called an **identity.**

An equation that is false for every value of the variable is called a **contradiction.**

3. Solve and then state whether the equation is conditional, an identity, or a contradiction.

a) $3(3 - 4x) = 4x - 2(8x - 3)$

$9 - 12x = 4x - 16x + 6$

$9 - 12x = -12x + 6$

$+12x \qquad +12x$

$9 \neq 6$

Contradiction

$\emptyset$

b) $-2x + 5 - 2x - 8 = 3 - 4(x + 1) - 2$

$-4x - 3 = 3 - 4x - 4 - 2$

$3 - 4x - 6$

$-4x - 3 = -4x - 3$

$+4x \qquad +4x$

$0 - 3 = -3$

$+3 \qquad +3$

$\boxed{0 = 0}$

Identity